Compaction of freshly pluviated granulates under uniaxial and multiaxial cyclic loading

Compaction de sable remblais sous un chargement uniaxial et multiaxial cyclique.

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ABSTRACT: For the prediction of the cumulative settlement of soils the information about the strain amplitude, average stress and the initial void ratio is not sufficient. Consideration of soil fabric is of crucial importance. The proposed explicit model for cyclic settlement incorporates a novel "back polarization" tensor which memorizes the history of cyclic deformation. Strain amplitude is redefined for multiaxial loading and rotation of principal directions. The experimental evidence is provided.

1 INTRODUCTION

A considerable displacement of structures may be caused by an accumulation of the irreversible deformation in the subsoil with increasing number of load cycles. Even relatively small amplitudes may significantly contribute. This can endanger the long-term serviceability of structures which have large cyclic load contributions and small displacement tolerances. Under undrained conditions similar phenomena may lead to an accumulation of pore water pressure, to soil liquefaction and eventually to a loss of overall stability. Displacements due to cyclic loading depend strongly on subtle state parameters which cannot be expressed by the customary state variables (stress and void ratio) only. A large amount of good quality experimental data and a sound theoretical framework are required for the description of cumulative soil behaviour.

2 EXPLICIT OR IMPLICIT APPROACH

From a numerical point of view, two computational strategies can be considered: an implicit and an explicit one. *Explicit* or *N-type* models, cf. Sagaseta (1991), are similar to creep laws wherein the number of cycles N is used instead of time. This formulation implements a direct estimation of strain accumulation due to a bunch of strain cycles of a given amplitude. The recoverable (resilient) part of the deformation is calculated in order to estimate the strain amplitude only. This amplitude is assumed constant within the bunch and the permanent (residual) deformation due to a package of cycles is calculated with direct ('explicit') empirical formulas. Several such formulas have been proposed for example by Martin (1975), Sawicki (1991), Suiker (1998). Here we use the model of Sawicki as reference and extend it using results from our numerical and laboratory tests.

Implicit models are general-purpose constitutive relations which reproduce each single load cycle with small strain increments. The accumulation of stress or strain appears as a by-product of this calculation, resulting from the fact that the loops are not perfectly closed. Despite obvious advantages of implicit formulations (flexibility, elegance) we have to resort to explicit algorithms for numerical reasons, cf. Niemunis (2000). Actually we shall use a *semi-explicit* procedure interrupting cyclic pseudo-creep in order to apply *control cycles* calculated implicitly.

3 REFERENCE MODEL

Sawicki and Świdziński (1989) basing on experimental results from their simple shear device, proposed an explicit model with a purely volumetric accumulation rule

$$\overset{\circ}{\mathbf{T}} = \mathsf{E} : (\mathbf{D} - \dot{\boldsymbol{\epsilon}}^{acc}) \quad \text{with} \quad \epsilon_{ij}^{acc} = -\frac{1}{3}\epsilon^{acc\ v}\delta_{ij}, \tag{1}$$

in which the volumetric strain was described with a so-called 'universal densification curve'

$$\epsilon^{acc\ v} = C_1 \ln\left[1 + C_2\tilde{N}\right], \quad \text{with} \quad \tilde{N} = \frac{1}{2}\|\boldsymbol{\epsilon}^{*\ ampl}\|^2 N\,. \tag{2}$$

$C_1(e_0), C_2(e_0)$ are parameters related to the initial void ratio e_0, and $\tilde{N}$ is the number of cycles N scaled with the square of the deviatoric strain amplitude $(\gamma^{ampl})^2 = \frac{1}{2}\|\boldsymbol{\epsilon}^{*\ ampl}\|^2$, wherein $2\boldsymbol{\epsilon}^{*\ ampl}$ is the largest difference between two points of the strain cycle measured in deviatoric strain space.

4 CONCLUSIONS FROM THE CURRENT RESEARCH

The reference explicit model could be easily implemented into the FE code as presented by Niemunis (2001). The numerical results as well as experimental data from numerous cyclic triaxial tests only partly confirmed the assumptions of the reference model. In particular the following observations have been made.

- It is the accumulation and not the number of cycles N that should be scaled by the square of the strain amplitude, i.e. $\epsilon^{acc}(n\epsilon^{ampl}, N \dots) = n^2\epsilon^{acc}(\epsilon^{ampl}, N, \dots)$ but the postulated unique function $\epsilon^{acc}(\tilde{N}, \dots)$ wherein $\tilde{N} = (\gamma^{ampl})^2 N$ could not be found, see Fig. 1a and 1b.

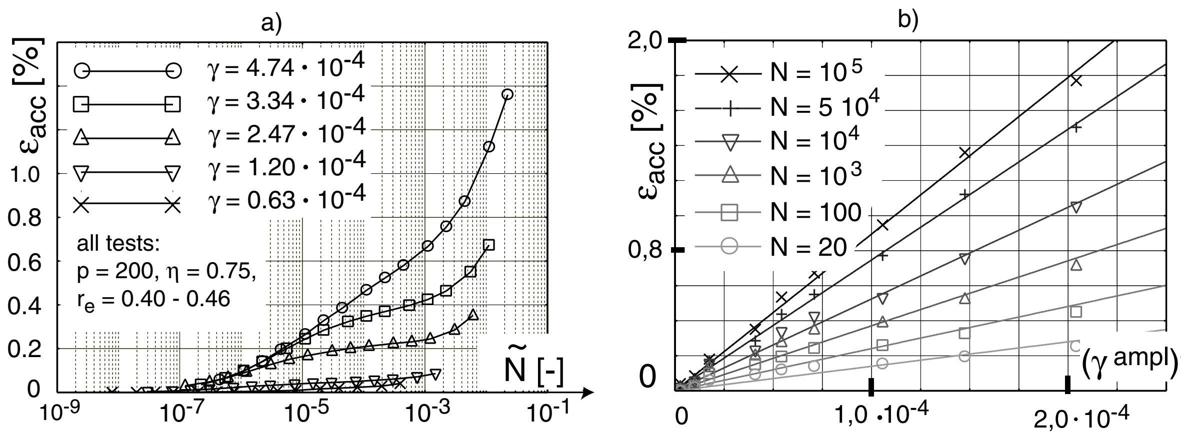

Figure 1. (a) Discrepancy from 'universal densification curve' in triaxial tests; (b) Accumulation as a second order homogeneous function of strain amplitude.

- The triaxial tests show that the influence of stress ratio η, pressure, void ratio and strain loop polarization cannot be neglected. The novel accumulation formula is proposed as

$$\epsilon^{acc} = C_1 \left(\frac{\epsilon^{ampl}}{\epsilon^{ampl}_{ref}}\right)^2 \left[\ln(1 + C_2 N) + C_3 N\right]\ f_Y(Y)\ f_r(e,p)\ f_\pi \tag{3}$$

with the reference amplitude $\epsilon^{ampl}_{ref} = \|\Delta_{max}\boldsymbol{\epsilon}\| = \frac{1}{\sqrt{2}}10^{-4}$ (or with $\gamma^{ampl}_{ref} = 10^{-4}$) and material constants C_i. The observed accumulation decreases significantly slower than the logarithmic function

(2) and therefore (2) is proposed to be supplemented with the linear term $C_3 N$. The functions f_Y, f_r and f_π are discussed further.

• The deviatoric strain cannot in general be disregarded and accumulated strain should be seen as a tensor

$$\boldsymbol{\epsilon}^{acc} = \epsilon^{acc}\,\mathbf{m} \qquad \text{with the unit tensor } \mathbf{m} \tag{4}$$

For the medium dense sand the flow rule has been found to lie along **m** normal to the modified Cam-clay

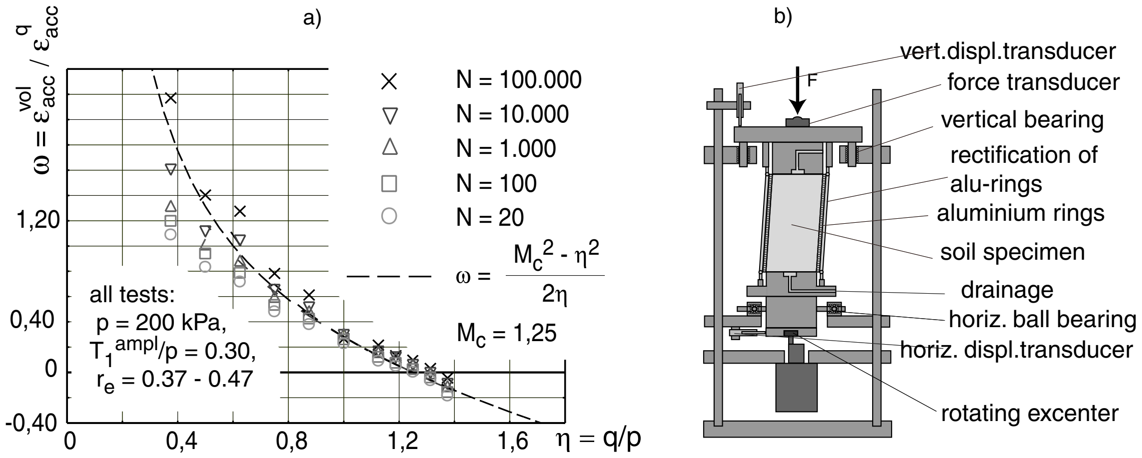

Figure 2. (a) Cyclic flow rule $\omega(\eta)$ b) cyclic multiaxial direct simple shear (CMDSS) device

surface, i.e.

$$\omega(\eta) = \frac{m_v}{m_q} = \frac{\operatorname{tr}\mathbf{D}^{acc}}{\sqrt{\frac{2}{3}}\|\mathbf{D}^{*acc}\|} \approx \frac{M^2 - \eta^2}{2\eta} \tag{5}$$

wherein $\eta = q/p = \sqrt{\frac{3}{2}}\|\mathbf{T}^*\|/(\frac{1}{3}\operatorname{tr}\mathbf{T})$, see Fig. 2a. The inclination of the CSL is given by $M = 6\sin\varphi/(3 - \sin\varphi)$. Such **m** turns out to be more accurate than the hypoplastic flow rule used earlier by Niemunis (2001). Unlike the dilatancy rule (e.g. by Rowe (1962)) for monotonic behaviour, the direction of cyclic flow has been found to depend on the void ratio, i.e. $\mathbf{m} = \mathbf{m}(\eta, e, p)$. Formulation of a specific formula, however, needs more experimental data than currently available.

• The rate of accumulation measured in triaxial tests increases strongly with the stress ratio η, especially if η is close to M. This dependence can be described using the well known yield function by Matsuoka and Nakai (1982). Defining

$$\bar{Y} = \frac{Y - 9}{Y_c - 9} \quad \text{with} \quad Y = -\frac{I_1 I_2}{I_3} = \frac{27(3 + \eta)}{(3 + 2\eta)(3 - \eta)} \quad \text{and} \quad Y_c = \frac{9 - \sin^2\varphi}{1 - \sin^2\varphi} \tag{6}$$

the observed accumulation rate increases according to

$$f_Y = \exp(C_4 \bar{Y}^2) \quad \text{with} \quad C_4 \approx 2\,. \tag{7}$$

The experimental results pertaining to this effect have been depicted in Fig.3b.

• The dependence of the accumulation rate on pressure and void ratio $f_r(e, p)$ cannot be captured using relative density $r_e = (e - e_d)/(e_c - e_d)$ wherein $e_d(p)$ and $e_c(p)$ are the minimum and critical void ratios. Keeping $r_e =$ const the rate of cyclic accumulation was observed to be almost proportional to p^{-1} (for $N = 10^5$), i.e. $\dot{\epsilon}^{acc}$ decreases(!) with p. This means that the critical state line $e_c(p)$ usually

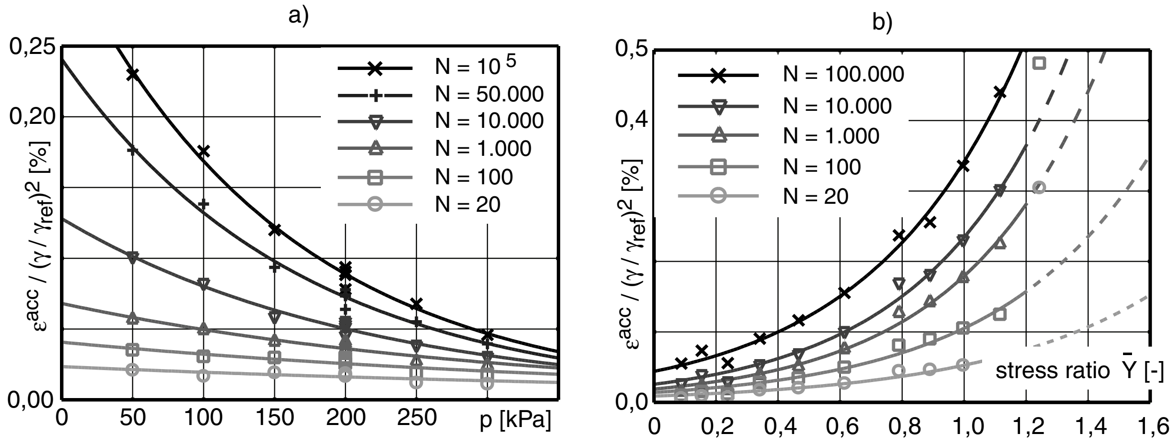

Figure 3. Rate of accumulation influenced by a) mean stress level p; b) stress ratio $\bar{Y}$

described by inclination λ in $e - \log p$ diagram does not apply to pairs (p, e) of identical cyclic accumulation rate of accumulation, see Fig.3a.

• Estimation of strain amplitude would be incomplete without a proper consideration of multiaxial loading, including polarization and shape of the strain loop. In the current study these effects were tested in a novel CMDSS device shown in Fig. 2b. In Fig. 4a a circular strain path with a diameter (size) $8 \cdot 10^{-3}$

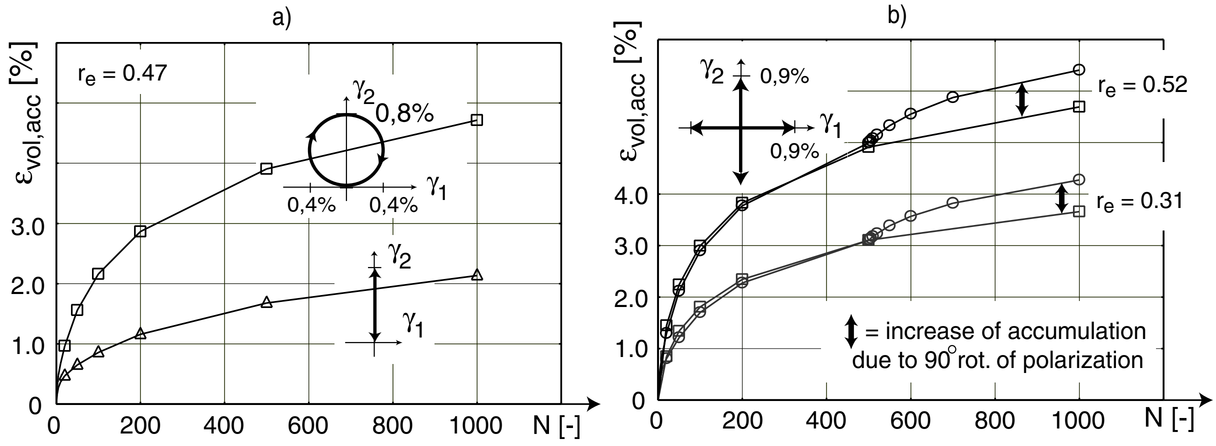

Figure 4. Rate of accumulation influenced by a) shape of the strain loop; b) rapid change of polarization

is shown to generate 2.5 times larger accumulation than an uniaxial strain path of the same size. This effect can be described with an extended definition of the deformation amplitude. A similar problem is known from the fatigue analysis of metals, cf. Ekberg (2000) or Papadopoulos (1994). Suppose that we are given a single strain loop understood as a sequence of discrete strain points $\boldsymbol{\epsilon}(t_i)$, $i = 1, \ldots, N$, not necessarily coaxial, in 6-D space. From the centre of the circumscribed hyper-sphere we perform a projection of the loop along the direction $\vec{\mathbf{r}}^{\,(6)}$ towards the most distant point of the loop. The superposed arrow $\vec{\sqcup}$ denotes the unit tensor obtained by normalization of $\sqcup$. We repeat this procedure until an uniaxial projection is obtained. The dyadic products $\vec{\mathbf{r}}^D \otimes \vec{\mathbf{r}}^D$ are used in the definition of amplitude as follows

$$\mathsf{A}_{\epsilon}{}^{P} = \sum_{D=1}^{6} P_D \vec{\mathbf{r}}^D \otimes \vec{\mathbf{r}}^D \quad \text{and} \quad \epsilon^{ampl} = \|\mathsf{A}_{\epsilon}{}^{P}\| \tag{8}$$

wherein P_D is the perimeter of D-dimensional projection. The above definition of amplitude has been explained in more detail by Niemunis (2003).

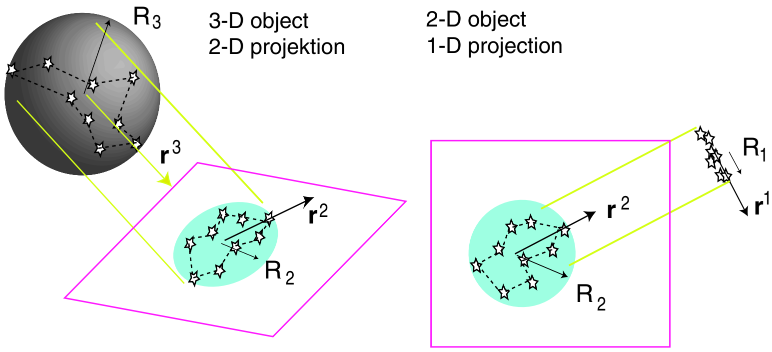

Figure 5. The directions $\mathbf{r}^D$ and the sizes R^D of the strain loop amplitude

• As shown in Fig. 4b, a rapid change of polarization causes a significant increase of accumulation rate. The term polarization is understand as the normalized amplitude $\vec{\mathsf{A}}_\epsilon = \mathsf{A}_\epsilon / \|\mathsf{A}_\epsilon\|$. It is memorized by the material model as a "back polarization" $\vec{\boldsymbol{\pi}}$ corresponding to the amplitude A_ϵ in the recent history. Consider a package of cycles with the amplitude $\mathsf{A}_\epsilon{}^{(1)}$ followed by another package with $\mathsf{A}_\epsilon{}^{(2)}$. If $\vec{\mathsf{A}}_\epsilon{}^{(1)} :: \vec{\mathsf{A}}_\epsilon{}^{(2)} = 1$ holds, the polarizations are identical and no additional increase of accumulation rate should appear. However, if $\vec{\mathsf{A}}_\epsilon{}^{(1)} :: \vec{\mathsf{A}}_\epsilon{}^{(2)} = 0$, the polarizations are perpendicular to each other and in such case the rate should be increased. Of course granulates are able to adapt themselves to a given polarization, i.e. $\vec{\boldsymbol{\pi}} \rightarrow \vec{\mathsf{A}}_\epsilon$. This adaptation is proposed to have the following form

$$\mathrm{d}\boldsymbol{\pi} \quad = \quad C_5 \left(\vec{\mathsf{A}}_\epsilon - \vec{\boldsymbol{\pi}} \right) \mathrm{d}N \quad \text{with} \quad C_5 > 0 \tag{9}$$

and the increase of the accumulation rate can be calculated from

$$f_\pi \quad = \quad 1 + C_6 (1 - \vec{\mathsf{A}}_\epsilon :: \vec{\boldsymbol{\pi}}) \, . \tag{10}$$

• The elastic stiffness for calculation of a single strain loop can be assumed proportional to $(p/p_{\text{atm}})^{2/3}$ as commonly used. The isotropic stiffness tensor, however, is not the best choice. The stiffness in vertical direction $\dot{T}_1/D_1$ turned out to increase (!) with η, except for the first cycle.

• The reference model describes accumulation as a function of the number of cycles N. However, the initial value N_0 which was assumed to be equal zero for freshly pluviated sand is difficult to estimate *in situ*. A simple analysis of the spatial stress fluctuations, see Triantafyllidis (2000), showed that smoothing thereof should occur spontaneously (lower energy) and that it should be accompanied by an increase of stiffness. Unfortunately an experimental attempt to correlate N_0 with the low strain stiffness in the RC-device failed, cf. Wichtmann (2001).

• As pointed out by Niemunis (2001), results from explicit models require a special space-integration algorithm in order to be independent of FE mesh

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6 BIBLIOGRAPHY

A. Ekberg (2000). Rolling contact fatigue of railway wheels. PhD thesis, Chalmers University of Technology. Solid Mechanics.

G.R. Martin, W.D.L. Finn, and H.B. Seed (1975). Fundamentals of liquefaction under cyclic loading. Journal of the Geotechnical Engineering Division ASCE, 101(GT5):423–439,

H. Matsuoka and T. Nakai (1982) . A new failure for soils in three-dimensional stresses. In Deformation and Failure of Granular Materials, pages 253–263, Proc. IUTAM Symp. in Delft.

A. Niemunis(2000). Akkumulation der Verformung infolge zyklischer Belastung des Bodens - numerische Strategien. In Th. Triantafyllidis, editor, Boden unter fast zyklischer Belastung: Erfahrungen und Forschungsergebnisse, volume 32, pages 1–20. Lehrstuhl für Grundbau und Bodenmechanik der Ruhr-Universität Bochum

A. Niemunis (2003). Extended hypoplastic models for soils. PhD thesis, Politechnika Gdańska, Habilitation, Monografia 34.

A. Niemunis and J. Helm (2001). Settlement of a strip foundation due to cyclic loading. centrifuge model and FE-calculation. In Soils Mechanics and Geotechnical Engineering, pages 761–764. Balkema, August Proceedings of 15-th International Conference in Istambul, Turkey.

I.V. Papadopoulos (1994) A new criterion of fatigue strength for out-of-phase bending and torsion of hard metals. International Journal of Fatigue, 16:377–384,

P. Rowe (1962). The stress-dilatancy relation for static equilibrium of an assembly of particles in contact. Proceedings of the Royal Society of London, 269:500–527,.

G. Sagaseta, V. Cuellar, and M. Pastor. (1991). Cyclic loading. In Deformation of soils and displacements of structures, volume 3, pages 981–999, Proceedings of the 10-th ECSMFE in Firenze, Italy.

A. Sawicki (1991). Mechanika gruntów dla obciązen cyklicznych. Wydawnictwo IBW PAN, Gdańsk, .

A. Sawicki and W. Świdziński (1989). Mechanics of sandy subsoil subjected to cyclic loadings. International Journal for Numerical and Analytical Methods in Geomechanics, 13:511–529.

A.S.J. Suiker (1998). Fatigue behaviour of granular materials. Technical Report 7-98-119-3, Delft University of Technology, Faculty of Civil Engineering.

Th. Triantafyllidis and A. Niemunis (2000) Offene Fragen zur Modellierung des zyklischen Verhaltens von nichtbindigen Böden. In Th. Triantafyllidis, editor, ”Boden unter fast zyklischer Belastung: Erfahrungen und Forschungsergebnisse”, pages 109–134. Ruhr-Universität Bochum (Eigenverlag) Heft 32.

T. Wichtmann, T. Sonntag, and T. Triantafyllidis (2001). Über das Erinnerungsvermögen von Sand unter zyklischer Belastung. Bautechnik, 78(12):852–865 .