Stochastic modelling of settlements due to cyclic loading for soil-structure interaction

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ABSTRACT: In numerous FE calculations of strip foundations on sand with randomly distributed (autocorrelated) properties we have shown that the ratio of the differential settlement to the mean settlement is about three times larger in case of cyclic loading than under monotonic loading. On the basis of an accumulation model it is explained why the structure under cyclic loading is more sensitive to the heterogeneities in the subsoil than under static loading. Some codes of practice restrict the differential settlements indirectly imposing limits on the mean settlements. This may lead to a dangerous underestimation of constrained stresses in the superstructures.

1 INTRODUCTION

A foundation settlement may lead to a considerable increase of internal forces in a structure and may affect its serviceability. For shallow foundations wider than 1m admissible settlements (serviceability of structure) rather than the bearing capacity usually dictate the design (Schmertmann 1970). The traditional methods, e.g. (Meyerhof 1965), of settlement prediction are based on elasticity theory and stiffness moduli measured in oedometric tests. Unfortunately, such predictions are not accurate enough. According to (Sievering 1979) the variation coefficient of the settlement ratio $s_{\text{calc}}/s_{\text{measured}}$ is about 50%. Comparative studies for the available methods (Jeyapalan and Boehm 1986; Wahls 1997) indicate inconsistent predictions of the magnitude of the settlement. Therefore in the past numerous empirical settlement prediction methods have been proposed. They are based on correlations between settlements and in-situ testing results. Most frequently these correlations are formulated for pressuremeter tests (Briaud and Jeanjean 1994), for cone penetration tests (Schmertmann 1970), standard penetration tests (Alpan 1964; Schultze and Sherif 1973), dilatometer modulus tests (EM-1110-1-1904 1990), seismic cone penetration tests (Fioravante, Jamiolkowski, Lo Presti, Manfredini, and Pedroni 1968; Mayne 2000), or for plate load tests (Bond 1961). They have been established under "usual" conditions (EM-1110-1-1904 1990). The popularity of the empirical methods is an obvious indicator of a poor performance of predictions based on elasticity (let alone the Winkler model). For example, such factors like the spatial variability of stiffness, its stress dependence, plastic effects at the edges of the foundation and small strain stiffness are often disregarded. The evidence of misery of the classical methods for shallow foundations is the fact that they have been outperformed by an artificial intelligence neural network (trained with 200 cases of actual measured settlements), cf. (Shahin and Maier 2000).

Although most structures can tolerate a substantial uniform settlement, it is often restricted by building codes (up to say 5 cm). Actually, it is the *differential* settlement that decisively affect the serviceability of the construction in the first line. In extreme cases distortions may endanger the structural integrity. The limits for differential settlements are sometimes expressed by the angular distortion being the differential settlement diminished by tilting. Depending on the type of construction the limiting angular distortion can vary between 1/125 and 1/2000.

It is believed that imposing limitations on total settlements the codes of practice restrict indirectly the *differences* of settlements. Also in this paper we follow the widely used suggestion (Skempton and MacDonald 1956; D'Appolonia, D'Appolonia, and Brissette 1968), that in order to limit the differential settlement the total settlement should not exceed some limit values. For example, according to (Terzaghi and Peck 1948), most buildings (with framed structures) can tolerate 20 mm differential settlements and since they are unlikely larger than 75% of the average total settlement, a safe guide for buildings is to restrict the settlements to 25 mm. According to (TM-5-818-1-AFM-88-3 1983) the indirect means of controlling the differential settlement on sand is to limit the total settlement to 4 cm.

Of course, several additional aspects should be considered in this context, in particular the susceptibility to settlements, e.g. of historical buildings or of buildings of prefabricated panels with weak fastenings, etc. The construction of the foundation (isolated pads, frames, rafts, ...), statically indeterminate design etc. can be of importance too. The limiting tensile strains (cracks become visible) are given in various recommendations, e.g. (Padfield and Sharrock 1983; EM-1110-1-1904 1990). In some cases the radius of the curvature of the deflection line is restricted, e.g. $R > 2$ km for mining regions, cf. (Smoltczyk 1990). Also the form of distortion may be relevant: an unreinforced masonry wall in hogging is twice more vulnerable than in sagging. The tilting is usually restricted by aesthetic criteria: excessing 0.25% it becomes visible. A famous example of tilting is the tower of Piza inclined by 10%.

In this paper we discuss the differential and total settlements of shallow foundations on sand due to *cyclic* loading. Unfortunately, sand deposits are usually much more heterogeneous than the clayey soils. As a result, differential settlements are likely to be higher in sand deposits than in clay profiles (Maugeri, Castelli, Massimino, and Verona 1998).

However, unlike clays, sands exhibit little or no delay in settlements due to creep or consolidation. This is beneficial because, the measured settlements during the construction phase allow for some precaution. Observing carefully the settlements during the construction period (under chiefly static loading) one can predict the future differential settlements and counterbalance them, e.g. by corrections in the geometry of the higher stores of the superstructure, by reinforcement of the construction or by soil improvement. In this way one can remedy a serious damage or an excessive deformation of the superstructures.

Settlements induced by cyclic loading are especially troublesome because they appear after the construction is finished and precautions to equilibrate settlements, e.g. by underpinning or compensation grouting ("Soilfrac" technology), are difficult. Obviously, repairs of under-use constructions are very expensive. Moreover, the differential settlement due to cyclic loading with respect to the average settlement turns out to be significantly larger than in the static case. This paper explains why a structure under cyclic loading is more sensitive to the heterogeneities in the subsoil than under static loading.

We investigate the differences in settlements using a fictitious subsoil with stochastically generated (and spatially correlated) distributions of void ratio and related soil properties. The differential settlements of sand due to high-cycle loading (i.e. a large number $N > 10^4$ of relatively small $\varepsilon^{\text{ampl}} < 10^{-3}$ cycles) are compared with the analogous differential settlement due to static loading. The predictions under both monotonous and cyclic loading are calculated for two strip foundations under plane strain conditions using the FE-method and suitable constitutive models (Niemunis and Herle 1997; Niemunis 2003; Niemunis, Wichtmann, and Triantafyllidis 2004a; Niemunis, Wichtmann, and Triantafyllidis 2004b).

Since the constitutive models have been formulated basing on relatively large sand samples (0.2×0.1 m) they comprise already a kind of "short range" averaging of properties within a sample. A usual sample consists of millions of Voronoi cells (each composed of several grains) and need not be stochastically analysed on the granular level, (Nübel 2002). Only the "long range" variability of soil parameters is of interest. By assumption, the spatial variability of the parameters is dictated by the distribution of the void ratio e. This distribution is assumed to be uniform between $e_{\max}$ and $e_{\min}$. The spatial correlation of the $e(\mathbf{x})$ field is discussed in Section 5.

2 VOID RATIO DISTRIBUTION

In natural sand layers the mechanical properties may vary strongly. Also this variability may change from site to site so there are no general rules except that the properties should be investigated at place by sounding. This leads to questions, about the properties to be tested and the necessary number of soundings. The answer is not easy and depends on many factors, which renders the soil engineering to be an art as well as the science. In this paper we will not solve this dilemma but merely demonstrate that cyclic loading increases (about three times) the probability of differential settlement.

The most important soil parameters depend on the effective stress $\mathbf{T}$ and on the void ratio e. They determine the stiffness (it increases with $(\operatorname{tr}\mathbf{T})^{0.7}$ but decreases with the stress obliquity $\hat{\mathbf{T}} = \mathbf{T}/\operatorname{tr}\mathbf{T}$) and the strength (dilatancy, peak friction angle). The constitutive models used here are rather complex and their comprehensive presentation is outside the scope of this paper. For monotonic loading we use a hypoplastic constitutive model with intergranular strain (Niemunis and Herle 1997; Niemunis 2003) and for cyclic loading a high-cycle model (Niemunis, Wichtmann, and Triantafyllidis 2004a; Niemunis, Wichtmann, and Triantafyllidis 2004b). Unfortunately, little or nothing is usually known about the spatial distribution of the stress and the void ratio in situ (in the horizontal direction). Therefore we restrict ourselves to the generation of stochastic fields of the void ratio only. They cause a fluctuation of strength and stiffness and eventually a fluctuation of stress. The soil stiffness shear G depends on the void ratio

$$G \sim \frac{(a-e)^2}{1+e}. \tag{1}$$

according to (Hardin and Black 1966) (with the material constant $a \approx 1.8$ for sand). This relatively small variability of stiffness leads to a variability of the initial stress by the application of the self weight of the soil. Note, however, that initially slight inhomogeneity of geostatic stress is growing as the self weight is being applied: the stress increments are proportional to stress itself (because the stiffness is a function thereof). This feedback effect amplifies the spatial stress fluctuations. We expect the resulting initial stress field to be more or less realistic. In a static analysis of settlements on spatially random soil (Paice, Griffith, and Fenton 1996) the variation coefficient of the Young modulus is cited to lie between 2% and 42% with a recommended value of 30%. Unfortunately, the temporal and spatial fluctuation of stress presented in the literature (Howell and Behringer 1997; Behringer and Miller 1997; Niemunis 2003) pertains usually to the level of grains, which is of little use in the present context.

3 MONOTONIC MODEL FOR SOILS

The hypoplastic constitutive model originates from the Karlsruhe research group, where it has been developed since the late seventies (Kolymbas 1978; Gudehus 1996; Wolffersdorff 1996). We use a recent version of the model (Niemunis 2003) which considers the improved shear stiffness (parameter ν). The Jaumann stress rate $\mathring{\mathbf{T}}$ predicted by the hypoplastic model can be written in the form

$$\mathring{\mathbf{T}} = f_b f_e \hat{\mathsf{E}}(\mathbf{D} - f_d \hat{\mathbf{m}} Y \|\mathbf{D}\|), \tag{2}$$

wherein the stiffness $\hat{\mathsf{E}}$, the flow direction $\hat{\mathbf{m}}$ (=unit tensor) and the degree of nonlinearity $0 < Y < 1$ are functions of stress obliquity $\hat{\mathbf{T}}$. The scalar barotropy factor f_b describes the increase of stiffness with the mean stress $p = -\mathrm{tr}\,\mathbf{T}/3$, the pyknotropy function f_e describes the decrease of stiffness with the void ratio e and f_d controls the peak friction angle depending on p and e. The critical state concept is implemented in the function f_d.

In order to implement the increased stiffness for the so-called "small strain" range the hypoplastic model (2) has been extended by adding an intergranular strain which is a tensor variable that memorizes the direction of straining in the recent deformation history of the material, cf. (Niemunis and Herle 1997). This part is of importance for the settlement predictions as well as for cyclic loading with a limited number of cycles.

The parameters used in the FE calculation (Section 6) are given in Table 1.

φ_c [°]	h_s [MPa]	n [-]	e_{d0} [-]	e_{c0} [-]	e_{i0} [-]	α [-]	β [-]
32.8	150	0.40	0.575	0.908	1,044	0.12	1.0

R [−]	m_R [−]	m_T [−]	χ [−]	β_R [−]	ν [−]
10^{-4}	6.5	3	6	0.1	0.2

Table 1: Parameters of the hypoplastic model for a fine sand (a sand commonly used in centrifuge tests in Bochum (Helm, Laue, and Triantafydillis 2000))

4 HIGH-CYCLE MODEL FOR SOILS

Recently (Niemunis, Wichtmann, and Triantafyllidis 2004a; Niemunis, Wichtmann, and Triantafyllidis 2004b) we have proposed a so-called *explicit*-type constitutive model for the calculation of settlements due to cyclic loading. It is a high-cycle model, i.e. packages of cycles (with a constant amplitude) are treated as increments similarly as a fatigue-type load. The accumulation of strain or stress is directly simulated. Treating the number of cycles N as a time-like variable the cumulative phenomena can be expressed using the viscoplastic formalism in which the strain accumulation per cycle $\mathbf{D}^{\mathrm{acc}}$ is treated as a viscous rate:

$$\mathbf{D}^{\mathrm{acc}} = \hat{\mathbf{m}} \left(\frac{\varepsilon^{\mathrm{ampl}}}{\varepsilon_{\mathrm{ref}}}\right)^2 f_N f_Y f_p f_e f_\pi \,. \tag{3}$$

The "direction" of strain accumulation is given by the flow rule $\hat{\mathbf{m}} = \mathbf{m}(\hat{\mathbf{T}})$. The intensity of accumulation is a function of the strain amplitude $\varepsilon^{\mathrm{ampl}}$ (the crucial dependence in the present context), of the average stress $\mathbf{T}$ (via f_Y and f_p), of the void ratio (via f_e), of the number cycles f_N (soil structure effect) and of the shape (via $\varepsilon^{\mathrm{ampl}}$) and the polarization of the amplitude (via f_π). Of course, loose sands show higher accumulation rates. Perhaps less obvious intuitively is the experimental finding that the rate of accumulation is higher at lower stress levels $\mathrm{tr}\,\mathbf{T}$,

$$f_p = \exp\left[-C_p \left(\frac{p^{\mathrm{av}}}{p_{\mathrm{atm}}} - 1\right)\right], \tag{4}$$

wherein $C_p \approx 0.43$ is a constant and $p_{\mathrm{atm}} = 100$ kPa. The amplitude is proposed to be described by a 4-th order tensor the scalar measure of which if $\varepsilon^{\mathrm{ampl}}$. This contains information about the polarization and the shape of the strain path within the cycle (for example cycles consisting in the rotation of the principal directions are automatically considered). The cyclic history is described by two parameters: the so-called back polarization tensor and the number of cycles weighted by their amplitudes. We skip further discussion of (3) and interested readers are referred to the detailed descrip-

C_{ampl} [-]	$\varepsilon_{\text{ref}}^{\text{ampl}}$ [-]	C_{N1} [-]	C_{N2} [-]	C_{N3} [-]
0.54	10^{-4}	$1.21\cdot10^{-3}$	0.39	$5.7\cdot10^{-5}$

C_p [−]	p_{ref} [kPa]	C_Y [−]	C_e [−]	e_{ref} [−]
0.43	100	2.0	0.52	0.908

Table 2: Parameters of the high cycle model for a fine sand (a sand commonly used in our centrifuge tests in Bochum, (Helm, Laue, and Triantafydillis 2000))

tion of the model (Niemunis, Wichtmann, and Triantafyllidis 2004a; Niemunis, Wichtmann, and Triantafyllidis 2004b) and of the experimental findings about the cyclic accumulation phenomena (Wichtmann, T. Niemunis, and Triantafyllidis 2004; Wichtmann, Niemunis, and Triantafyllidis 2004; Wichtmann, Niemunis, and Triantafyllidis 2005). Depending on the boundary conditions the accumulation of both stress (= 'cyclic pseudo-relaxation') and deformation (='cyclic pseudo-creep') may result according to

$$\overset{\circ}{\mathbf{T}} = \mathsf{E} : (\mathbf{D} - \mathbf{D}^{\text{acc}} - \mathbf{D}^{\text{pl}})\,. \tag{5}$$

The plastic strain rate $\mathbf{D}^{\text{pl}}$ is necessary to prevent excessively large stress obliquities in the soil.

The parameters used in the FE calculations are given in Table 2.

5 RANDOM MODELLING OF SOIL

Various properties of soil may be considered as random. In the present work, the void ratio e is of primary interest. Its scatter over a certain volume of soil is modelled by random fields discretized on the FE mesh.

In order to generate a random field of void ratio we need the estimates of its characteristic values, eg. the mean, the variance and the spatial correlation. Such estimates may be extracted from in-situ measurements. For example, the mean of the respective parameter of a known sample of n measurements reads

$$E[\sqcup] = \frac{1}{n}\sum_{i=1}^{n} \sqcup_i\,. \tag{6}$$

Alternatively a maximum likelihood estimator (MLE) could be used, (DeGroot and Beacher 1993; Fenton 1999). The estimator should be *unbiased* (the estimated average of a population is equal to the mean of the measured sample) and *consistent* (the dispersion decreases with the size of a sample i.e. the number of measurements).

First let us examine the void ratio e at a given point $\mathbf{x}$ in space. For a fictitious subsoil considered here, the mean void ratio is assumed to be $\bar{e} = 0.8$ and the probability density function (PDF) is chosen to be constant and equal to $1/0.4$ over the e-range from 0.6 to 1.0. We do not use the normal Gaussian distribution here because extremely small or extremely large void ratios may cause problems in the constitutive models. Moreover, negative void ratios are physically impossible.

Let us suppose, we have a statistical sample of n measurements of e taken at various locations $\mathbf{x}_i$ with $i = 1,\ldots,n$ and the void ratio at a point $\mathbf{x}_i$ is related to the void ratio at a point $\mathbf{x}_j$. For all m pairs (i,j) of points $\mathbf{x}_i$ $\mathbf{x}_j$ of a given statistical sample (set of n measurements) lying at a given distance d we may quantify the difference of the void ratios. Furthermore, let this difference be isotropic, i.e. independent of the orientation of the vector $\mathbf{x}_i - \mathbf{x}_j$. Having measured the void ratio $e(\mathbf{x})$ over some area we can distinguish its trend $\bar{e}(\mathbf{x})$ (a mean value, often estimated by linear regression) and scatter $e'(\mathbf{x})$:

$$e(\mathbf{x}) = \bar{e}(\mathbf{x}) + e'(\mathbf{x}). \tag{7}$$

For simplicity, we assume that the void ratio does not decrease with the depth, i.e. the trend to be equal to the mean, $\bar{e}(\mathbf{x}) = 0.8$. The autocorrelation of the scatter $e' = e - \bar{e}$ could be evaluated from measurements using the moment estimate of the isotropic autocovariance of e'. For a typical pair of points at distance d it is

$$\rho(e'(\mathbf{x}),d) = \frac{1}{m}\sum_{i}^{n}\sum_{j}^{n} e'(\mathbf{x}_i)e'(\mathbf{x}_j)w(\mathbf{x}_i,\mathbf{x}_j), \tag{8}$$

wherein $w(\mathbf{x}_i,\mathbf{x}_j) = \begin{cases} 1 & \text{if } \|\mathbf{x}_i - \mathbf{x}_j\| \approx d \\ 0 & \text{otherwise} \end{cases}$, n is the total number of points and $m(d) = \sum_i^n \sum_j^n w(\mathbf{x}_i,\mathbf{x}_j)$.

For our fictitious subsoil we assume the Markovian (exponential) spatial correlation function between the void ratios e' at points $\mathbf{x}_i$ and $\mathbf{x}_j$ at distance $d = \|\mathbf{x}_i - \mathbf{x}_j\|$ according to

$$\rho(e',d) = \exp\left(-d/\theta\right), \tag{9}$$

wherein θ denotes the correlation length.

It is not easy to determine the correct value of θ for a given type of soil because soil properties may vary at many scales (fractally). Using the estimations of θ from the literature one should select a correlation length estimated on a similar soil over a domain of a similar size (Fenton and Griffith 2002). Of course, we have to choose a value between the limit cases $\theta \to 0$ and $\theta \to \infty$ for which the void ratios are either fully uncorrelated (statistically independent) or perfectly correlated, respectively. In these both extremes, the differential settlement Δs will vanish. (Fenton and Griffith 2002) give some recommendations for the choice of θ: In the problem with two footings at distance D, the assumption $\theta = D$ would be conservative. Similarly for a single foundation of the width B the correlation scale $\theta = B$ would be also conservative.

Instead, we decided to try out three correlation lengths $\theta = 0.5, 2.0$ and 20.0m. Fortunately, as we shall see, the central conclusion of this paper is scarcely dependent on θ.

The differential settlement is calculated with the FE method using quadrilateral elements with four Gauss integration points each. Various possibilities of random field discretization are reported in the literature (Matthies, Brenner, Bucher, and Soares 1997). For simplicity reasons, we discretize the random field of void ratios e by the FE mesh using the mentioned Gauss points.

The field variability is described by the following isotropic autocorrelation function:

$$C_{ij} = \sigma^2 \exp\left(-\frac{d_{ij}}{\theta}\right) \tag{10}$$

with

$$d_{ij} = \|\mathbf{x}_i - \mathbf{x}_j\|, \tag{11}$$

$$\sigma = \frac{1}{2}(e_{\max} - e_{\min}), \tag{12}$$

and $e_{\max} = 1.0$, $e_{\min} = 0.6$.

In the numerical example presented in Section 6 we are using 1098 elements with 4 Gauss points per element, that results in a real symmetric covariance matrix (4392×4392).

This matrix $\mathbf{C}$ is then transformed to the uncorrelated space by the orthogonal transformation (spectral decomposition) based on its eigenvalues $\Lambda = \mathrm{diag}\{\lambda_1, \ldots, \lambda_n\}$ and an orthogonal matrix Φ,

$$\mathbf{C} = \Phi \cdot \Lambda \cdot \Phi^T. \tag{13}$$

The matrix Φ is composed of orthonormalized eigenvectors (in columns, numbering same as for λ-s), $\Phi = \{\Phi_1, \ldots, \Phi_n\}$.

According to (Novak, Lawanwisut, and Bucher 2001), only a limited number $n_a < n$ of the eigenvalues and eigenvectors Λ, Φ are necessary to restore basic features of the uncorrelated covariance matrix $\mathbf{C}$ with acceptable accuracy. In the present work, we obtain satisfactory results with $n_a = 500$.

The field $e'(\mathbf{x})$ is generated by using the first n_a largest eigenvalues and the corresponding eigenvectors Φ_i, $i = 1, \ldots, n_a$ multiplied by random factors, viz.

$$e'(\mathbf{x}) = \sum_{i=1}^{n_a} r_i^{[-1,1]} \frac{1}{2} \sqrt{\lambda_i} \Phi_i \tag{14}$$

wherein $r^{[-1,1]}$ is a uniform variate (random real number with constant PDF) from the range $[-1,1]$ and λ_i is the i-th eigenvalue of $\mathbf{C}$. Of course one may easily find $r^{[-1,1]} = 2r^{[0,1]} - 1$ using the intrinsic random function $r^{[0,1]}$ provided in all programming languages. The final field of void ratios is obtained from (7).

Although a set of N_{sim} fields (=simulations) $e'(\mathbf{x})$ could be generated directly using (14), with random $r^{[-1,1]}$ (= Monte Carlo method) a small improvement is yet proposed. Instead of the afore mentioned uniform variate $r^{[-1,1]}$ in (14), we apply a so-called Latin Hypercube Sampling (LHS), (Florian 1992).

For each eigenvector i the domain $[-1,1]$ is divided into N_{sim} equal intervals, e.g. we use $N_{\text{sim}} = 10$ and number the intervals (-1,-0.8), ..., (0.8,1) with $k = 1, \ldots 10$. A random permutation (=sequence of ks) is generated for each eigenvector i and the midpoint values of the intervals are set to $r^{[-1,1]}$ in (14). For example, Fig. 1, if the random permutation for the i-th eigenvector were (7,3,1,...) then the values (0.3, -0.5, -0.9, ...) would be used in place of $r^{[-1,1]}$ in (14) in the first simulations. In a more general

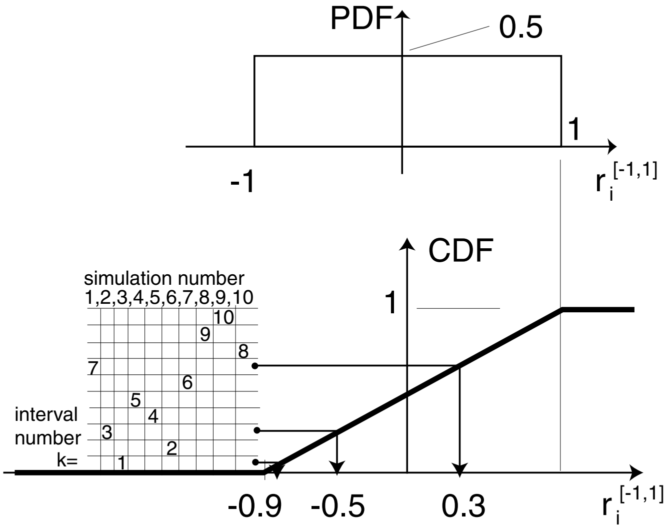

Figure 1: *Latin Hypercube Sampling.*

case of a nonuniform PDF we choose the intervals in such way, that the probabilities of the void ratio to lie within each interval k are equal. The LHS is reported to be more effective than the direct Monte-Carlo method, especially in combination with nonlinear finite element analysis, (Novak, Lawanwisut, and Bucher 2001). Advantages of the Monte-Carlo method follow from the fact that this sampling covers uniformly the entire range of variability of a variable of interest.

Although the multipliers $r_i^{[-1,1]} \lambda_i$ (treated individually) have uniform PDFs their sum (14) has not. However, this is not a serious shortcoming, because we do not know the exact PDF anyway. An unfavorable effect is that sporadically some values $e(\mathbf{x})$ may lie slightly outside of the prescribed scope $[e_{\min}, e_{\max}]$, which may cause problems in the constitutive equations of the soil. Therefore we have to check and correct a few peak values manually.

6 NUMERICAL ANALYSIS

In this section we investigate a plane strain boundary value problem with two strip foundations of the width 1m each at the axial distance of 6m, Fig. 2. For the FE calculation the commercial programm ABAQUS is adopted with user material routine, user initial stress and user initial state variables procedures.

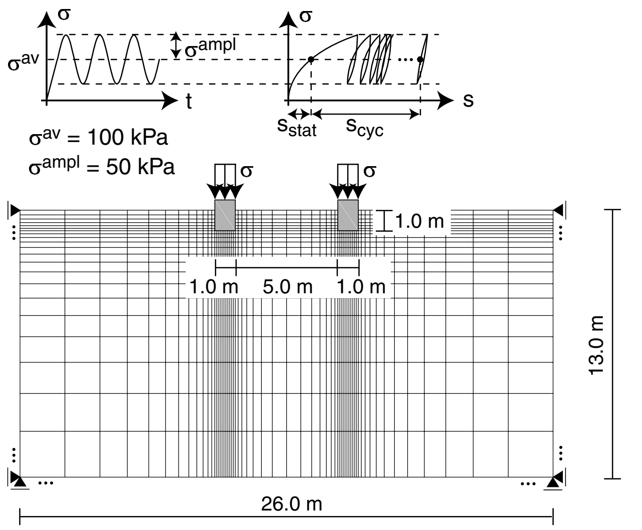

Figure 2: *BVP and FE mesh for two strip foundations*

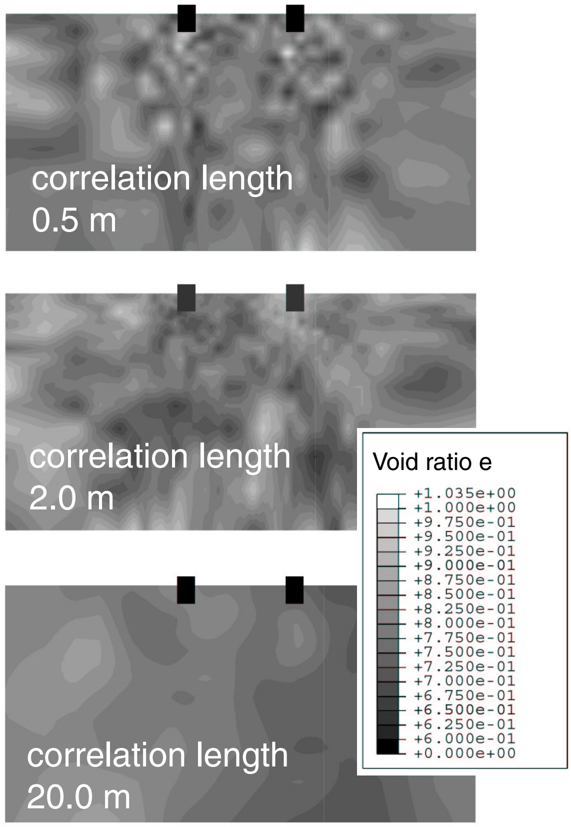

Figure 3: *Void ratio distributions generated using different correlation lengths θ.*

Using the method described in the previous sections 30 stochastic fields of the void ratio with the corresponding fluctuations of the initial stress have been generated. We try out three correlation lengths $\theta = 0.5, 2.0$ and 20 m and generate 10 fields per correlation lengths. Examples of such fields are presented in Fig. 3.

Although equally loaded (at first monotonically and then cyclically) the foundations exhibit a differential settlement Δs which for each calculation is normalized by the mean settlement $\bar{s}$. The x-coordinate of each point in Fig. 4 is the ratio $\Delta s/\bar{s}$ for the pair of foundations under monotonic loading and the y-coordinate presents the analogous ratio $\Delta s/\bar{s}$ under cyclic loading (after 10^5 cycles) calculated with the same material constants and using the same field of void ratio. It can be seen from Fig. 4 that the settlement $\bar{s}$ due to cyclic loading is accompanied by a three times larger differential settlement Δs than in the static case. An explanation of this effect lies in the nature of cyclic accumulation in soils. The essential point is that the cyclic accumulation is proportional to the square of the strain amplitude, see Eq. 3, whereas the static settlement is approximately proportional to the load i.e. to the amplitude. Therefore cyclic accumulation is a short range phenomenon (involves the soil volume only in the vicinity of the foundation) and the static load has a larger range (the active zone in analytical calculation of settlement is about three to four times the width of the foundation). The probability of finding an extreme dense zone of sand under one foundation and an extreme loose zone under the other one is therefore higher in the case of cyclic loading.

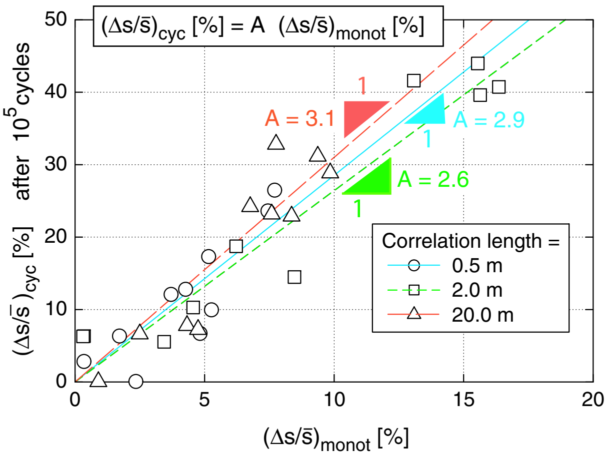

Figure 4: *Indirect limitation of differential settlement via total settlement should be three times more restrictive in case of cyclic loads. This result seems to be independent of the correlation length θ.*

7 CONCLUSION

A series of FE calculations of settlements of a pair of strip foundations on a random (spatially correlated) subsoil is performed applying monotonous and cyclic loading.

From the comparisons of the differential settlement under cyclic and static loading its increased sensitivity to cyclic loading is demonstrated. A large number, say $> 10^4$, of relatively small load cycles lead to differential settlements Δs which are about three times larger then in the static case with the same mean settlements $\bar{s}$. In other words, the ratios $\Delta s/\bar{s}$ under monotonous loading were three times smaller compared to analogous ratios caused by cyclic loading. The prediction of settlements of structures due to high-cycle loading requires therefore a thorough testing program in-situ and a good quality constitutive model for the predictions.

The settlements due to cyclic loading should be considered as a long term soil structure interaction problem, and should not be underestimated in the life-time prediction of the respective structure. Admittedly, the presented results do not include the compensatory effect of the stiffness of the superstructure and as such do not consider this interaction. However, if the calculation of settlements were combined with the static analysis of the structure then (in the statically indeterminate case) the differential settlements would be smaller at the cost of increased internal forces in the structure. In consequence, the life-time expectations of the structure would be reduced. The study of this effect is underway within the Cooperative Research Centre SFB 398 in the cooperation between the projects A8 and C1. Apart from the increased differential settlements due to the small size of the active zone under cyclic loading, the rate of cyclic accumulation increases (paradoxically, cf. Section 4) under foundations with smaller soil pressures which leads to a positive feedback effect. This also increases the differential settlements. The internal forces are therefore expected to grow during the cyclic loading speeding up the deterioration processes or even causing some plastic hinges in the structure.

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