

# PREDICTION OF DRAINED AND UNDRAINED CYCLIC BEHAVIOUR OF A FINE SAND USING A HIGH-CYCLE ACCUMULATION MODEL

Torsten WICHTMANN<sup>i)</sup> Benjamin ROJAS <sup>ii)</sup> Andrzej NIEMUNIS<sup>iii)</sup>  
Theodor TRIANTAFYLLIDIS<sup>iv)</sup>

## ABSTRACT

The paper presents an experimental study in order to quantify the isotropic hypoelastic stiffness tensor  $\mathbf{E}$  of the high-cycle accumulation (HCA) model proposed by Niemunis et al. [1].  $\mathbf{E}$  interrelates stress and strain accumulation rates and not stress and strain increments. A pressure-dependent bulk modulus  $K$  and a constant Poisson's ratio  $\nu$  are assumed. The bulk modulus of medium-dense fine uniform sand was determined from 12 pairs of drained and undrained cyclic triaxial tests. It is demonstrated that, within the investigated range,  $K$  does not depend on the amplitude-pressure-ratio. The bulk modulus measured for the fine sand is of similar magnitude as the  $K$ -values obtained in an earlier study on a medium coarse sand. The drained and undrained cyclic tests were recalculated using the HCA model. The good agreement between predicted and experimental data is demonstrated in the paper.

Keywords: high-cycle accumulation model, stiffness  $\mathbf{E}$ , drained and undrained cyclic triaxial tests

## INTRODUCTION

The high-cycle accumulation (HCA) model proposed by Niemunis et al. [1] can be used for the prediction of settlements or stress relaxation (e.g. excess pore water pressure accumulation) in non-cohesive soils due to a large number ( $N > 10^3$ ) of cycles with relative small strain amplitudes ( $\epsilon^{\text{ampl}} < 10^{-3}$ ) (so-called high- or polycyclic loading). The HCA model may be applied for example to foundations of on- and offshore wind power plants, to machine foundations or to foundations subjected to traffic loading. The HCA model is based on an extensive laboratory testing program with drained cyclic triaxial and multiaxial DSS tests (Wichtmann [3]).

The experimental study presented in the paper is dedicated to the isotropic hypoelastic stiffness tensor  $\mathbf{E}$  in the basic constitutive equation of the HCA model:

$$\dot{\boldsymbol{\sigma}}' = \mathbf{E} : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^{\text{acc}} - \dot{\boldsymbol{\epsilon}}^{\text{pl}}) \quad (1)$$

In Eq. (1) the superposed dot means a rate with respect to the number of cycles  $N$  (i.e. an increment per cycle). The colon denotes a double contraction (Gibbs notation).  $\boldsymbol{\sigma}'$  is the rate

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<sup>i)</sup>Research assistant, Institute of Soil Mechanics and Rock Mechanics, Karlsruhe Institute of Technology, Karlsruhe, Germany, e-mail: torsten.wichtmann@kit.edu

<sup>ii)</sup>Student, Universidad de Santiago de Chile, Santiago, Chile, e-mail: benjamin.rojasf@gmail.com

<sup>iii)</sup>Research assistant, Institute of Soil Mechanics and Rock Mechanics, Karlsruhe Institute of Technology, Karlsruhe, Germany, e-mail: andrzej.niemunis@kit.edu

<sup>iv)</sup>Professor and Director, Institute of Soil Mechanics and Rock Mechanics, Karlsruhe Institute of Technology, Karlsruhe, Germany, e-mail: theodoros.triantafyllidis@kit.edu

of effective stress,  $\dot{\boldsymbol{\varepsilon}}$  the strain rate,  $\dot{\boldsymbol{\varepsilon}}^{\text{acc}}$  the prescribed rate of strain accumulation and  $\dot{\boldsymbol{\varepsilon}}^{\text{pl}}$  a plastic strain rate preventing stress paths from passing the yield surface. The rate of strain accumulation is calculated as a product of a scalar intensity of accumulation  $\dot{\varepsilon}^{\text{acc}}$  and a direction of accumulation  $\mathbf{m}$  (unit tensor):

$$\dot{\boldsymbol{\varepsilon}}^{\text{acc}} = \dot{\varepsilon}^{\text{acc}} \mathbf{m} \quad (2)$$

The intensity of accumulation is calculated as the product of six functions each considering a separate influencing parameter:

$$\dot{\varepsilon}^{\text{acc}} = f_{\text{ampl}} f_e f_p f_Y \dot{f}_N f_\pi \quad (3)$$

The functions  $f_{\text{ampl}}$ ,  $f_e$ ,  $f_p$  and  $\dot{f}_N$  describing the influences of strain amplitude, void ratio, average mean pressure and cyclic preloading are introduced in the section on the test results. The function  $f_Y$  capturing the dependence of  $\dot{\varepsilon}^{\text{acc}}$  on average stress ratios  $\eta^{\text{av}} = q^{\text{av}}/p^{\text{av}}$  and the function  $f_\pi$  considering the effect of polarization changes are unimportant here because the tests were performed with isotropic average stresses ( $\eta = 0 = \text{const.}$ , i.e.  $f_Y = 1$ ) and with a constant polarization ( $f_\pi = 1$ ).

Depending on the boundary conditions (e.g. stress or strain control in element tests), Eq. (1) predicts a change of the average stress ( $\dot{\boldsymbol{\sigma}}' \neq \mathbf{0}$ ) and/or an accumulation of residual strain ( $\dot{\boldsymbol{\varepsilon}} \neq \mathbf{0}$ ). The stiffness  $\mathbf{E}$  in Eq. (1) is important for boundary value problems showing a significant stress relaxation due to cyclic loading (e.g. pile foundations). At present an isotropic hypo-elastic tensor is used for  $\mathbf{E}$ . Therefore, two elastic constants, e.g. bulk modulus  $K$  and Poisson's ratio  $\nu$  have to be determined.

In elastoplastic models  $\mathbf{E}$  would be determined experimentally from a small unloading (at the absence of plastic strain  $\Delta\boldsymbol{\varepsilon}^{\text{pl}}$ ), comparing the stress increment  $\Delta\boldsymbol{\sigma}'$  with the strain increment  $\Delta\boldsymbol{\varepsilon}$ :

$$\Delta\boldsymbol{\sigma}' = \mathbf{E} : (\Delta\boldsymbol{\varepsilon} - \underbrace{\Delta\boldsymbol{\varepsilon}^{\text{pl}}}_{\mathbf{0}}) \quad (4)$$

This procedure is not applicable to HCA models.  $\mathbf{E}$  has to be determined from cyclic tests, in particular from a comparison of cyclic creep and cyclic relaxation tests performed with the same average stress and density.

A first experimental study on  $K$  has been documented by Wichtmann et al. [5]. The pressure-dependent bulk modulus of medium dense, medium coarse uniform quartz sand (mean grain size  $d_{50} = 0.55$  mm, coefficient of uniformity  $C_u = d_{60}/d_{10} = 1.8$ , obtained from a sand pit near Dorsten, Germany) was quantified from 15 pairs of drained and undrained cyclic triaxial tests and described by:

$$K = A p_{\text{atm}}^{1-n} p^n \quad (5)$$

with the reference pressure  $p_{\text{atm}} = 100$  kPa and with constants  $A = 467$  and  $n = 0.46$ . However, due to the coarse grains the undrained test data had to be corrected with respect to membrane penetration effects. The method proposed by Tokimatsu et al. [2] was applied. An inaccuracy of the parameters  $A$  and  $n$  proposed for Eq. (5) may be caused by uncertainties associated with this correction.

Therefore, a new experimental study has been performed on a fine uniform quartz sand ( $d_{50} = 0.14$  mm,  $C_u = 1.5$ , obtained from a sand pit near Ludwigshafen, Germany) for which membrane penetration effects are negligible. The present paper reports on the experimental results of this study.

**EXPERIMENTAL DETERMINATION OF  $K$  AND  $\nu$** 

The bulk modulus  $K$  can be obtained from a pair of drained and undrained cyclic tests with similar initial conditions and with similar cyclic loading. In the axisymmetric case, Eq. (1) can be rewritten with Roscoe's invariants:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_v - \dot{\varepsilon}^{\text{acc}} m_v \\ \dot{\varepsilon}_q - \dot{\varepsilon}^{\text{acc}} m_q \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} m_v \\ m_q \end{bmatrix} = \frac{1}{\sqrt{\frac{1}{3}(p - \frac{q^2}{M^2 p})^2 + 6(\frac{q}{M^2})^2}} \begin{bmatrix} p - \frac{q^2}{M^2 p} \\ 2\frac{q}{M^2} \end{bmatrix} \quad (7)$$

with mean pressure  $p = (\sigma'_1 + 2\sigma'_3)/3$ , deviatoric stress  $q = \sigma'_1 - \sigma'_3$ , volumetric strain  $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$  and deviatoric strain  $\varepsilon_q = 2/3(\varepsilon_1 - \varepsilon_3)$ .  $\sigma'_1$ ,  $\sigma'_3$  and  $\varepsilon_1$ ,  $\varepsilon_3$  are the axial and the horizontal effective stress or strain components, respectively.  $G$  is the shear modulus,  $m_v$  and  $m_q$  are the volumetric and the deviatoric portions of the flow rule  $\mathbf{m}$ , respectively. Omitting  $\dot{\varepsilon}^{\text{pl}}$  in Eq. (1) is legitimate for homogeneous stress fields if the cyclic loading is not overlaid by monotonic loading. The critical stress ratio  $M$  is defined as

$$M = \begin{cases} M_c & \text{for } \eta \geq 0 \\ (1 + \eta/3)M_c & \text{for } M_e < \eta < 0 \\ M_e & \text{for } \eta \leq M_e \end{cases} \quad (8)$$

$$\text{with } M_c = \frac{6 \sin \varphi_c}{3 - \sin \varphi_c} \quad \text{and} \quad M_e = -\frac{6 \sin \varphi_c}{3 + \sin \varphi_c} \quad (9)$$

wherein  $\varphi_c$  is the critical friction angle and  $\eta = q/p$ . For isotropic average stress conditions ( $q = 0$ ,  $\dot{q} = 0$ ,  $m_q = 0$ ), Eq. (6) takes either the form of isotropic relaxation

$$\dot{p} = -K \dot{\varepsilon}^{\text{acc}} m_v \quad \leftrightarrow \quad \dot{u} = K \dot{\varepsilon}^{\text{acc}} m_v \quad (10)$$

under undrained conditions ( $\dot{\varepsilon}_v = 0$ ) or the form of volumetric creep

$$\dot{\varepsilon}_v = \dot{\varepsilon}^{\text{acc}} m_v \quad (11)$$

under drained conditions ( $\dot{p} = 0$ ). Comparing these equations one may eliminate  $\dot{\varepsilon}^{\text{acc}} m_v$  and obtain

$$K = \dot{u}/\dot{\varepsilon}_v \quad (12)$$

Therefore, the bulk modulus  $K$  can be calculated from Eq. (12) with the rate  $\dot{u}$  of pore pressure accumulation obtained from an undrained cyclic test and with the rate  $\dot{\varepsilon}_v = \dot{\varepsilon}_v^{\text{acc}}$  of volumetric strain accumulation from a drained cyclic test with similar initial conditions and with similar cyclic loading.

Poisson's ratio  $\nu$  can be determined from the evolution of the average effective stress in an undrained cyclic triaxial test with constant strain amplitude commenced at an anisotropic initial stress (see the example in Figure 1). For  $\dot{\varepsilon}_v = 0$  and  $\dot{\varepsilon}_1 = 0$  and therefore  $\dot{\varepsilon}_q = 0$  one obtains:

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} K & 0 \\ 0 & 3G \end{bmatrix} \begin{bmatrix} -\dot{\varepsilon}^{\text{acc}} m_v \\ -\dot{\varepsilon}^{\text{acc}} m_q \end{bmatrix} \quad (13)$$

The ratio of the relaxation rates  $\dot{q}/\dot{p}$ , that means the inclination of the average effective stress path depends on  $\nu$ :

$$\frac{\dot{q}}{\dot{p}} = \frac{3G m_q}{K m_v} = \frac{9(1 - 2\nu)}{2(1 + \nu)} \frac{1}{\omega} \quad \leftrightarrow \quad \nu = \frac{9 - 2\omega(\dot{q}/\dot{p})}{18 + 2\omega(\dot{q}/\dot{p})} \quad (14)$$

with the strain rate ratio

$$\omega = \frac{\dot{\varepsilon}_v^{\text{acc}}}{\dot{\varepsilon}_q^{\text{acc}}} = \frac{m_v}{m_q} = \frac{M^2 - (\eta^{\text{av}})^2}{2\eta^{\text{av}}} \quad (15)$$

The index "av" denotes the average value during a cycle. An experimental study on Poisson's ratio  $\nu$  for the HCA model has been presented by Wichtmann et al. [6]. For fine sand, a value of  $\nu = 0.32$  works well in most cases.

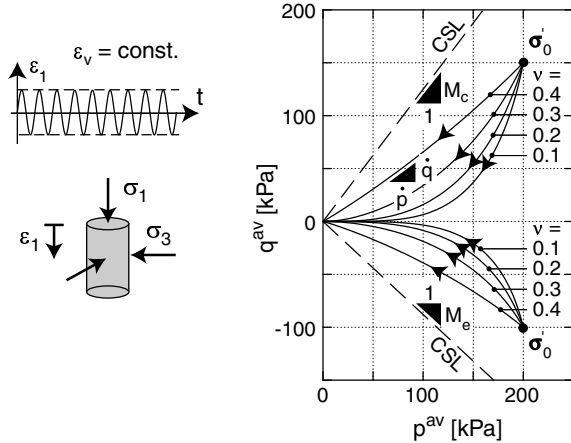


Fig. 1: Average effective stress paths in undrained cyclic triaxial tests with constant strain amplitude predicted by the HCA model for different Poisson's ratios  $\nu$

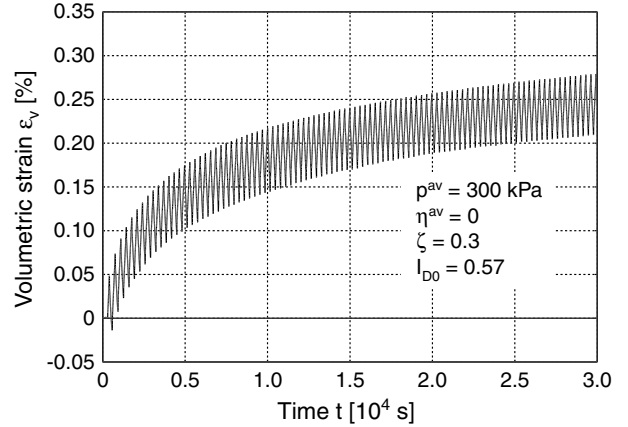


Fig. 2: Volumetric strain versus time in a drained cyclic triaxial test

## SAMPLE PREPARATION AND TESTING PROCEDURE

Twelve pairs of drained and undrained cyclic triaxial tests were performed. The cylindrical samples (diameter  $d = 10$  cm, height  $h = 10$  cm) were prepared by air pluviation and subsequently saturated with de-aired water. The two samples of a test pair were prepared with similar initial void ratios and consolidated under the same isotropic effective stress. The two samples were loaded with the same deviatoric stress amplitude, one under drained and the other one under undrained conditions.

All samples were medium dense. Four different initial effective mean pressures ( $p_0 = 50, 100, 200$  and  $300$  kPa) were tested. For each pressure, three test pairs with different amplitude-pressure ratios  $\zeta = q^{\text{ampl}}/p_0 = 0.2, 0.25$  or  $0.3$ , respectively, were performed. The loading was applied with a constant displacement rate of  $0.02$  mm/min (strain rate  $\dot{\varepsilon}_1 \approx 0.02$  %/min).

In all tests (drained and undrained) the first cycle was applied drained due to the following reasons. The first cycle may be *irregular* and may generate much more deformation than the subsequent *regular* ones. The HCA model predicts the accumulation due to the regular cycles only. In numerical calculations with the HCA model the first cycle is calculated using a conventional incrementally nonlinear constitutive model (see Niemunis et al. [1]). Since the initial conditions at the beginning of the regular cycles were intended to be similar in the drained and in the undrained tests, the first cycle was applied drained in both types of tests.

## TEST RESULTS

### Drained tests

The data from the drained tests can be used for both, the quantification of bulk modulus  $K$  and the determination of the parameters used in the functions  $f_{\text{ampl}}$ ,  $f_e$ ,  $f_p$  and  $f_N$  of the HCA

Parameter	$\varphi_c$	$C_{\text{ampl}}$	$C_e$	$e_{\text{ref}} = e_{\text{max}}$	$C_p$	$C_{N1}$	$C_{N2}$	$C_{N3}$
Value	$33.1^\circ$	1.40	0.55	1.054	0.23	$1.22 \cdot 10^{-4}$	0.64	0

Table 1: HCA model parameters for the fine sand

model.

In accordance with Eq. (15), a significant accumulation of volumetric strain was observed in the drained cyclic tests (see the example in Figure 2), while the deviatoric strain accumulation was negligibly small. The left-hand side of Figure 3 presents the measured accumulation curves  $\varepsilon_v^{\text{acc}}(N)$ . For a certain average mean pressure  $p^{\text{av}}$ , the accumulation rate increased with increasing stress amplitude. This is also evident in Figure 4a, where the residual strain after  $N = 10, 100$  and  $1000$  cycles is plotted versus a mean value  $\bar{\varepsilon}^{\text{ampl}} = 1/N \int \varepsilon^{\text{ampl}}(N) dN$  of the strain amplitude. The strain amplitude slightly decreased with  $N$ , because the tests were performed with a constant stress amplitude. On the ordinate of Figure 4a the residual volumetric strain has been divided by the void ratio function  $\bar{f}_e$  of the HCA model in order to purify the data from slightly different initial void ratios and different compaction rates:

$$f_e = \frac{(C_e - e)^2}{1 + e} \frac{1 + e_{\text{ref}}}{(C_e - e_{\text{ref}})^2} \quad (16)$$

wherein  $e_{\text{ref}} = e_{\text{max}}$  is a reference void ratio and  $C_e$  is a material parameter (see Table 1). The bar over  $\bar{f}_e$  in Figure 4a indicates that the void ratio function has been calculated with a mean value  $\bar{e} = 1/N \int e(N) dN$  of void ratio. Data for larger amplitude-pressure-ratios  $\zeta = 0.35$  and  $0.40$  was supplemented in the diagram for  $p^{\text{av}} = 200$  kPa. The amplitude function

$$f_{\text{ampl}} = (\varepsilon^{\text{ampl}} / \varepsilon_{\text{ref}}^{\text{ampl}})^{C_{\text{ampl}}} \quad (17)$$

with  $\varepsilon_{\text{ref}}^{\text{ampl}} = 10^{-4}$  was fitted to the data in Figure 4a delivering the parameter  $C_{\text{ampl}}$  for different  $p^{\text{av}}$ - and  $N$ -values. A mean value  $C_{\text{ampl}} = 1.4$  (see Table 1) is used in the following.

Some additional tests with different initial densities were performed in order to determine the parameter  $C_e$  of  $f_e$ . The intensity of strain accumulation increases with increasing void ratio (Figure 4b). A curve-fitting of  $f_e$  to the data in Figure 4b delivered a mean value  $C_e = 0.55$ . Since  $f_{\text{ampl}}$  is necessary to purify the data in Figure 4b (due to slightly different strain amplitudes) and  $f_e$  is used on the ordinate in Figure 4a, the determination of  $C_{\text{ampl}}$  and  $C_e$  was done iteratively.

The barotropy function

$$f_p = \exp[-C_p (p^{\text{av}} / p_{\text{ref}} - 1)] \quad (18)$$

with a reference pressure  $p_{\text{ref}} = 100$  kPa describes the increase of the accumulation rate with decreasing pressure. Its parameter  $C_p$  was obtained from a curve-fitting of Eq. (18) to the data plotted in Figure 4c. It shows the residual volumetric strain after different numbers of cycles, divided by  $\bar{f}_{\text{ampl}}$  and  $\bar{f}_e$ , as a function of the average mean pressure. The division by  $\bar{f}_{\text{ampl}}$  is necessary because the strain amplitude considerably increases with increasing pressure due to the constant amplitude-pressure-ratio  $\zeta$ . Three diagrams for the different  $\zeta$ -values of  $0.2, 0.25$  and  $0.30$  are given in Figure 4c. A mean value  $C_p = 0.23$  was obtained from the curve-fitting and is used in the following.

The parameters  $C_{N1} = 1.22 \cdot 10^{-4}$ ,  $C_{N2} = 0.64$  and  $C_{N3} = 0$  were determined from a curve-fitting of the function

$$\sqrt{3} f_N = \sqrt{3} C_{N1} [\ln(1 + C_{N2} N) + C_{N3}] \quad (19)$$

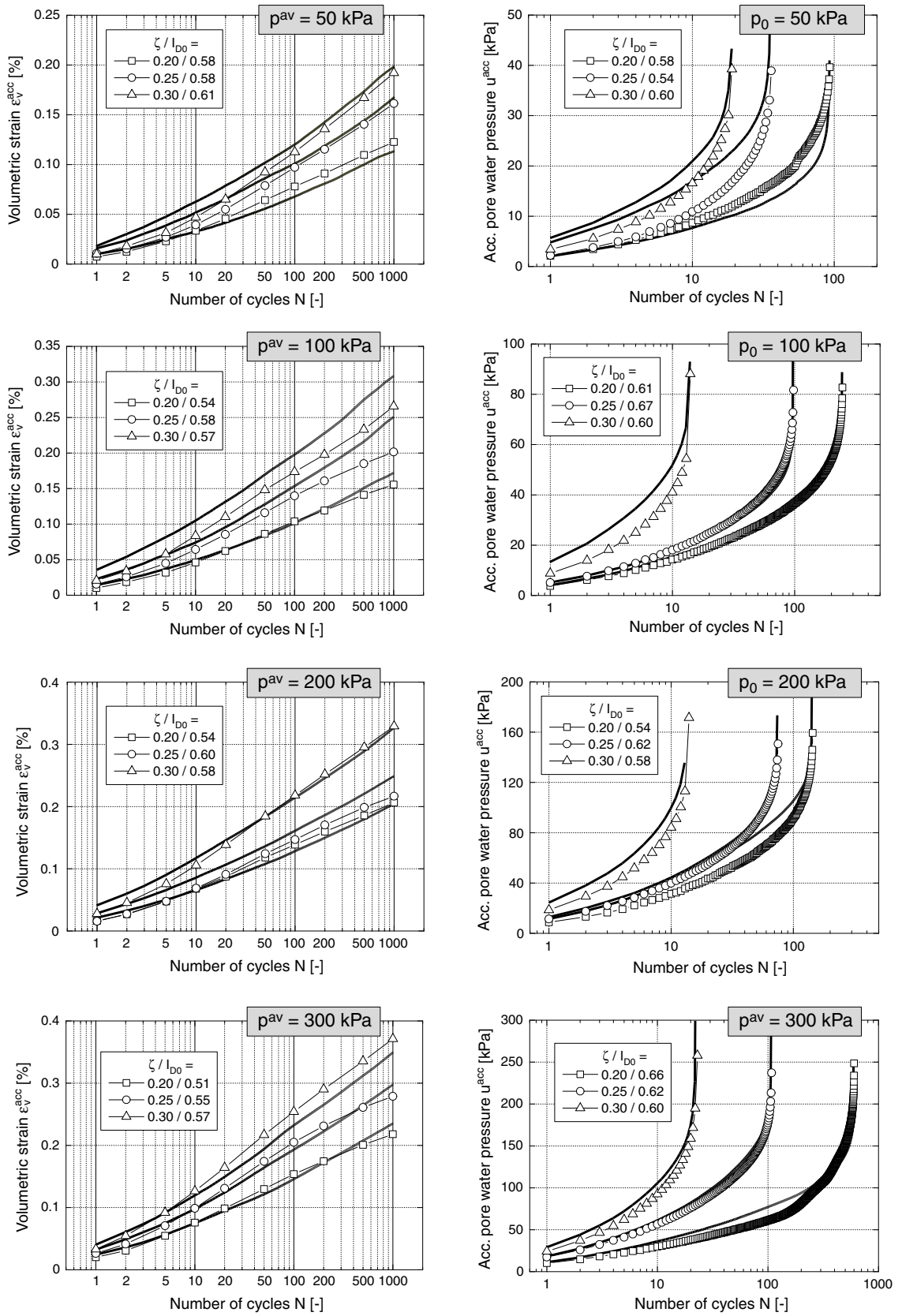


Fig. 3: Accumulated volumetric strain in the drained tests (left-hand side) and accumulated excess pore water pressure in the undrained tests (right-hand side) versus number of cycles, initial relative density  $I_{D0} = (e_{max} - e_0)/(e_{max} - e_{min})$  with  $e_{max} = 1.054$  and  $e_{min} = 0.677$

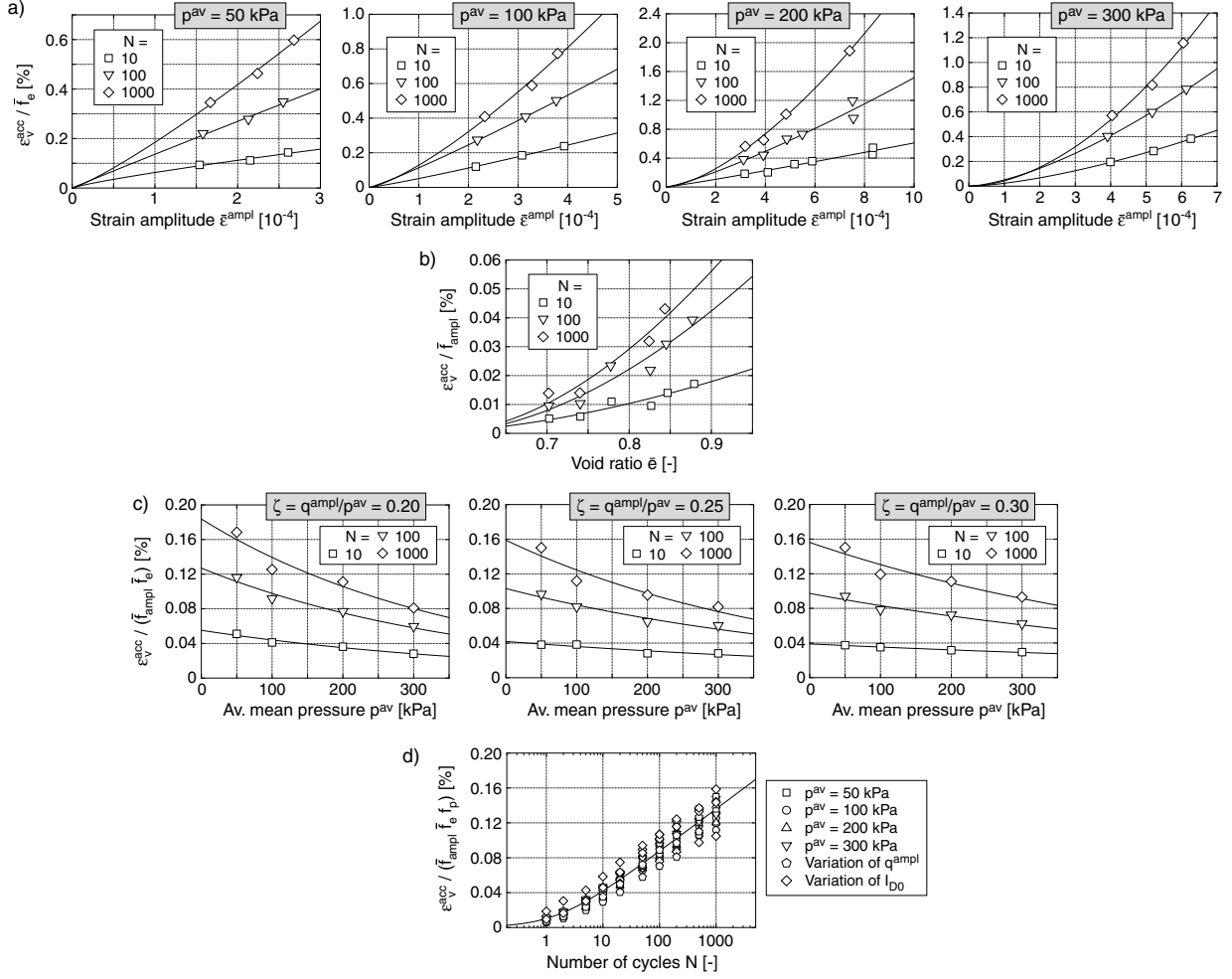


Fig. 4: Volumetric strain accumulated in the drained cyclic tests as a function of a) strain amplitude, b) void ratio, c) average mean pressure and d) number of cycles.

to the data in Figure 4d. In that figure, the accumulation curves  $\varepsilon_v^{acc}(N)$  were divided by  $\bar{f}_{ampl}$ ,  $\bar{f}_e$  and  $f_p$  in order to purify the data from the influences of amplitude, void ratio and pressure. For isotropic stress conditions,  $\varepsilon_v^{acc}(N) = \sqrt{3}\varepsilon^{acc}(N)$  holds (compare the flow rule of the HCA model, Eq. (15)). The function  $f_N$  in Eq. (19) represents the integral of the function  $\dot{f}_N$  of the HCA model

$$\dot{f}_N = C_{N1} \left[ C_{N2} \exp\left(-\frac{g^A}{C_{N1} f_{ampl}}\right) + C_{N3} \right] \quad (20)$$

which describes the decrease of the accumulation rate with increasing cyclic preloading, quantified by the historiotropic variable  $g^A$ :

$$g^A = \int f_{ampl} \left[ C_{N1} C_{N2} \exp\left(-\frac{g^A}{C_{N1} f_{ampl}}\right) \right] dN \quad (21)$$

For uniform sands, the parameter  $C_{N3}$  influences the rate of strain accumulation for large numbers of cycles ( $N > 10^4$ , Wichtmann et al. [4]) only. Therefore,  $C_{N3}$  was set to zero here.

The HCA model with the parameters summarized in Table 1 was used for recalculations of the drained cyclic tests. The initial void ratios and the measured strain amplitudes  $\varepsilon^{ampl}(N)$  were used as input. The thick solid curves in the diagrams on the left-hand side of Figure 3 were obtained from these recalculations. The experimental data and the prediction by the HCA model agree quite well.

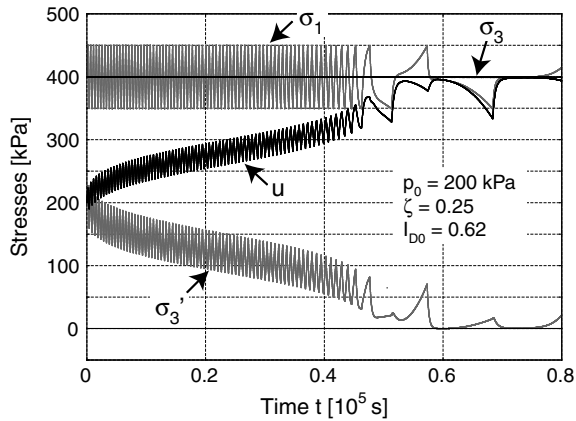


Fig. 5: Total and effective stress components versus time in an undrained cyclic triaxial test

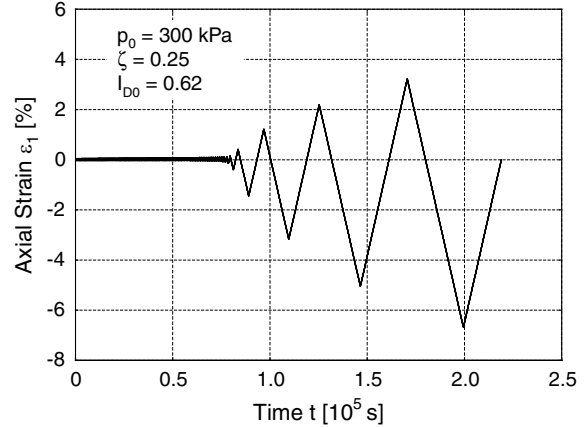


Fig. 6: Axial strain versus time in an undrained cyclic triaxial test

### Undrained tests

An accumulation of pore water pressure was observed in the undrained cyclic tests (see the example in Figure 5). After "initial liquefaction" (when  $\sigma'_1$  and  $\sigma'_3$  became temporarily zero for the first time) the axial strain amplitude increased considerably with each subsequent cycle (Figure 6). In Figure 7, the effective stress paths from all twelve undrained cyclic tests are presented in the  $p$ - $q$ -plane.

The curves of the excess pore water pressure versus the number of cycles are given on the right-hand side of Figure 3. For a given initial pressure, the diagrams show the well-known increase of the rate of pore water pressure accumulation  $\dot{u}$  with increasing amplitude-pressure-ratio  $\zeta = q^{\text{ampl}}/p_0$ .

### Bulk modulus from a comparison of drained and undrained cyclic test data

For the twelve pairs of drained and undrained cyclic tests the bulk modulus  $K$  was calculated from Eq. (12). The rate  $\dot{u} \approx \Delta u / \Delta N$  of pore water pressure accumulation was obtained from the undrained tests while the rate  $\dot{\varepsilon}_v^{\text{acc}} \approx \Delta \varepsilon_v^{\text{acc}} / \Delta N$  of volumetric strain accumulation was derived from the corresponding drained test data. In contrast to the study documented by Wichtmann et al. [5], the undrained test data were not corrected by membrane penetration effects since these effects are negligible for the tested fine sand.

In ideal case all state variables (average stress, density, cyclic preloading, strain amplitude) should be identical in the drained and in the undrained test. Otherwise Eq. (12) does not hold. However, despite similar values of the initial effective mean pressure  $p_0$ , initial void ratio  $e_0$  and deviatoric stress amplitude  $q^{\text{ampl}}$ , the conditions in the undrained and in the drained cyclic tests diverged with increasing number of cycles. In the undrained tests the average effective mean pressure  $p^{\text{av}}$  decreased and the strain amplitude  $\varepsilon^{\text{ampl}}$  increased. In the drained tests the void ratio and the strain amplitude slightly decreased. In order to calculate  $K$  from Eq. (12) it is important to evaluate  $\dot{u}$  and  $\dot{\varepsilon}_v^{\text{acc}}$  for exactly the same state, that means same values of  $\varepsilon^{\text{ampl}}$ ,  $e$ ,  $p^{\text{av}}$  and cyclic preloading. The possible discrepancies must be corrected by a factor  $f_c$  consisting of four multipliers:

$$f_c = \frac{f_{\text{ampl}}^{\text{UD}} f_e^{\text{UD}} f_p^{\text{UD}} \dot{f}_N^{\text{UD}}}{f_{\text{ampl}}^{\text{D}} f_e^{\text{D}} f_p^{\text{D}} \dot{f}_N^{\text{D}}} \quad (22)$$

The indices "UD" and "D" indicate the undrained or drained test, respectively. The functions  $f_{\text{ampl}}$ ,  $f_e$ ,  $f_p$  and  $\dot{f}_N$  of the HCA model have been introduced above. They were calculated with



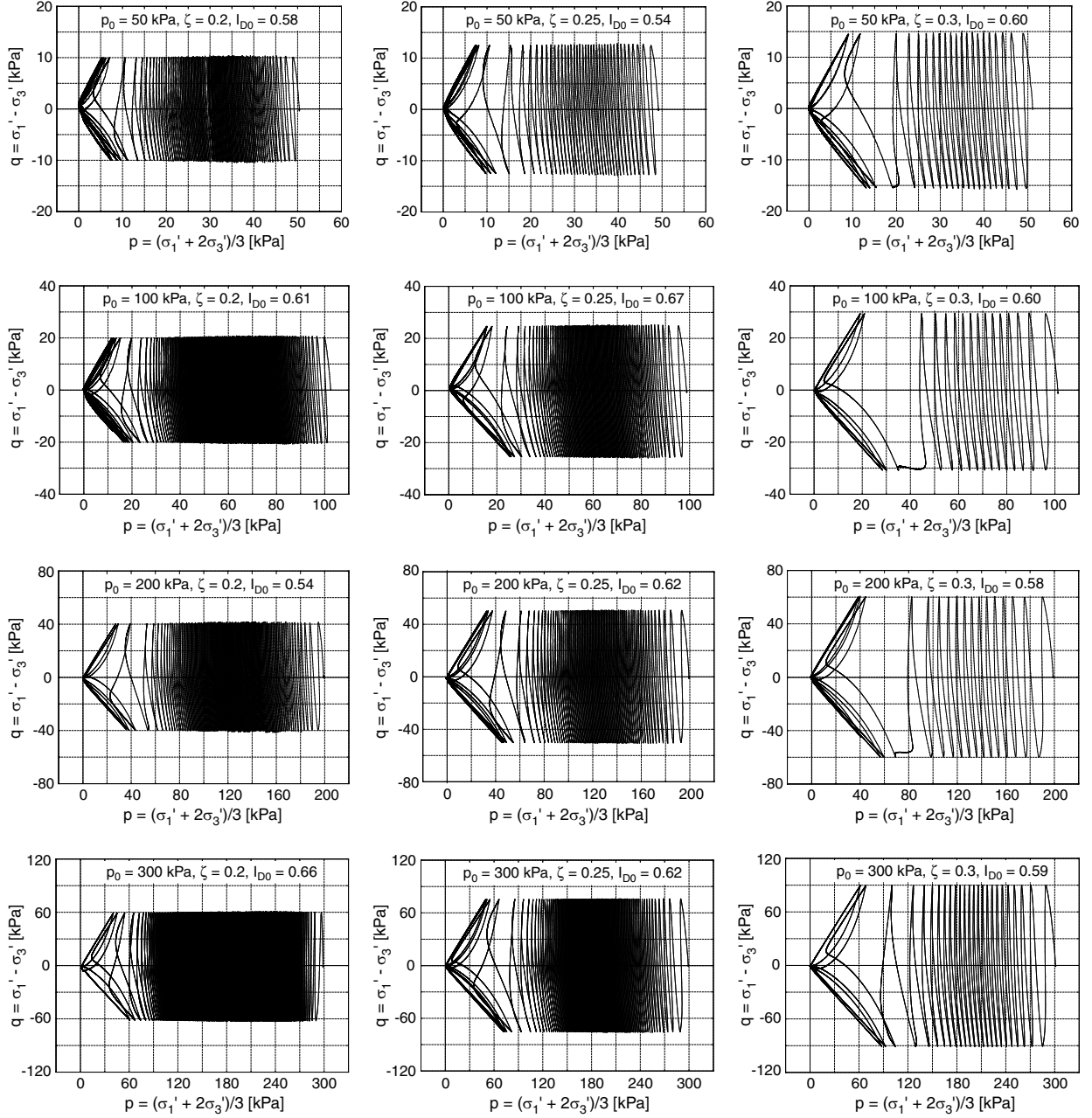


Fig. 7: Effective stress paths measured in the twelve undrained cyclic triaxial tests

the parameters given in Table 1. The volumetric strain accumulation rates  $\dot{\epsilon}_v^{\text{acc}}$  measured in the drained tests have been multiplied by  $f_c$  before  $K$  was evaluated from Eq. (12).

The bulk modulus  $K$  was determined for increments of the pore water pressure of  $\Delta u \approx 10$  kPa except the tests with  $p_0 = 50$  kPa where  $\Delta u \approx 5$  kPa was used. The data of  $K$  versus  $p^{\text{av}}$  is shown in Figure 8, where  $p^{\text{av}}$  is the average effective mean pressure in the undrained tests. The cloud of data in Figure 8 can be approximated by Eq. (5) with  $A = 440$  and  $n = 0.50$  (solid line in Figure 8). No significant influence of the amplitude-pressure-ratio  $\zeta$  could be found. The relationship  $K(p)$  obtained for the medium coarse sand (Wichtmann et al. [5]) has been added as dashed line in Figure 8. It demonstrates that the bulk moduli of the fine sand and the medium coarse sand are of similar magnitude. However, it should be kept in mind that the  $K$ -values derived for the medium coarse sand may be somewhat inaccurate due to the membrane penetration correction.

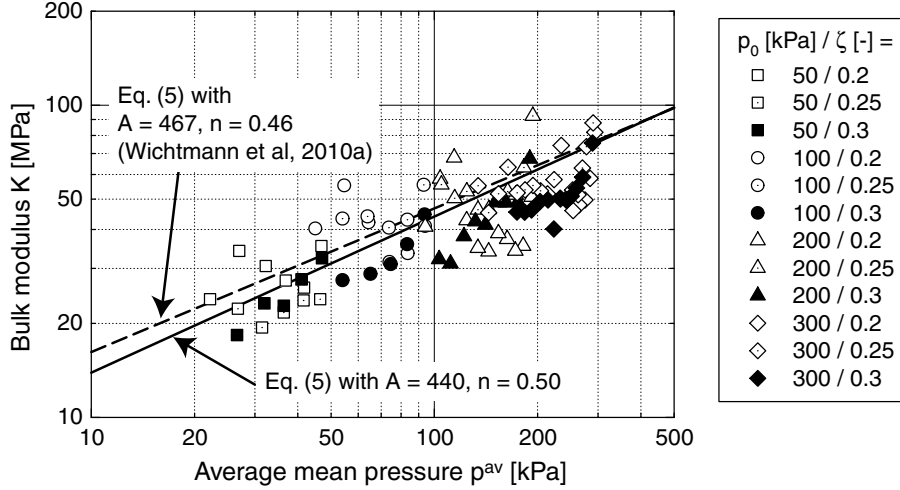


Fig. 8: Bulk modulus  $K = \dot{u}/\dot{\varepsilon}_v^{\text{acc}}$  as a function of average mean pressure  $p^{\text{av}}$ , obtained from twelve pairs of drained and undrained cyclic triaxial tests

The HCA model with the parameters in Table 1 and with the bulk modulus calculated from Eq. (5) with  $A = 440$  and  $n = 0.50$  has been used for recalculations of the undrained cyclic tests. The initial void ratios and the measured strain amplitudes  $\varepsilon^{\text{ampl}}(N)$  were used as input. The predicted curves of excess pore water pressure versus number of cycles have been added in the diagrams on the right-hand side of Figure 3 (thick solid curves). A good agreement between the HCA model prediction and the experimental data can be noticed.

It can be concluded that the HCA model with a single set of parameters, and with the bulk modulus determined according to the method described in the paper, describes well both, the strain accumulation under drained conditions and the accumulation of excess pore water pressure in the undrained case.

## SUMMARY, CONCLUSIONS AND OUTLOOK

The isotropic hypoeastic stiffness tensor  $\mathbf{E}$  of the high-cycle accumulation model proposed by Niemunis et al. [1] has been inspected in cyclic triaxial tests on a fine sand. The bulk modulus  $K$  was quantified from twelve pairs of drained and undrained cyclic tests. The assumption of a constant Poisson's ratio has been examined in another study (Wichtmann et al. [6]).

The two samples of a test pair were prepared with similar initial void ratios and consolidated under the same effective isotropic stress. The two samples were loaded with the same deviatoric stress amplitude, one under drained and the other one under undrained conditions. The bulk modulus  $K = \dot{u}/\dot{\varepsilon}_v^{\text{acc}}$  was calculated with the rate  $\dot{u}$  of pore water pressure accumulation from the undrained test and the rate  $\dot{\varepsilon}_v^{\text{acc}}$  of volumetric strain accumulation from the drained test.

Four different initial mean pressures  $p_0$  and three different amplitude-pressure-ratios  $\zeta = q^{\text{ampl}}/p_0$  were tested. The pressure-dependence of  $K$  is well described by Eq. (5) with constants  $A = 440$  and  $n = 0.50$ .  $K$  seems not to depend on amplitude. The bulk moduli obtained for the fine sand are of similar magnitude as those measured for a medium coarse sand (Wichtmann et al [5]).

In future, the dependence of bulk modulus  $K$  on the void ratio and on the average stress ratio  $\eta^{\text{av}} = q^{\text{av}}/p^{\text{av}}$  will be inspected experimentally. Furthermore, it is intended to develop a simplified determination procedure for  $K$  based on oedometric test data.

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