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ESTIMATION OF THE SMALL-STRAIN STIFFNESS OF GRANULAR SOILS TAKING INTO ACCOUNT THE GRAIN SIZE DISTRIBUTION CURVE

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ABSTRACT

Approximately 650 resonant column (RC) tests with additional P-wave measurements have been performed on 64 specially mixed grain size distribution curves of a quartz sand. The tested materials had different mean grain sizes d_{50}, coefficients of uniformity $C_u = d_{60}/d_{10}$ and fines contents FC. In a first test series, grain size distribution curves with a linear shape in the semi-logarithmic scale were studied. The RC tests revealed that the small-strain shear modulus $G_{\max}$ and the constrained elastic modulus $M_{\max}$ are independent of d_{50}, but strongly decrease with increasing C_u and FC. Furthermore, the test results showed that Hardin's equation with its commonly used constants can significantly over-estimate the small-strain shear modulus of well-graded granular soils. Therefore, this empirical equation has been extended by the influence of C_u and FC. A similar set of equations has been developed for $M_{\max}$. Correlations for the modulus degradation factor $G(\gamma)/G_{\max}$ are also proposed. A second test series was performed with piecewise linear, gap-graded, S-shaped and other smoothly shaped grain size distribution curves. It is demonstrated that the new correlations work well also for these "more complicated" grain size distribution curves.

Keywords: small-strain stiffness, granular material, grain size distribution curve, resonant column tests

INTRODUCTION

For feasibility studies, preliminary design calculations or final design calculations in small projects dynamic soil properties are often estimated by means of empirical formulas. The secant shear modulus G is usually described as a product of its maximum value $G_{\max}$ at very small shear strain amplitudes $\gamma < 10^{-6}$ and a modulus degradation factor $F(\gamma)$, i.e. $G = G_{\max}F(\gamma)$. A widely used empirical formula for the small strain shear modulus $G_{\max}$ of sand has been proposed by Hardin and Richart [4] and Hardin and Black [1] which is given in its dimensionless form here:

$$G_{\max} = A\frac{(a-e)^2}{1+e}(p_{\text{atm}})^{1-n}p^n \tag{1}$$

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with void ratio e, mean pressure p and atmospheric pressure $p_{\text{atm}} = 100$ kPa. The constants $A = 690$, $a = 2.17$ and $n = 0.5$ for round grains, and $A = 320$, $a = 2.97$ and $n = 0.5$ for angular grains were recommended by Hardin and Black [1] and are often used for estimations of $G_{\max}$-values for various sands. Hardin and Drnevich [2] proposed the following function for the modulus degradation factor $F(\gamma)$:

$$F(\gamma) = \frac{1}{\frac{\gamma}{\gamma_r}\left[1 + a\exp\left(-b\frac{\gamma}{\gamma_r}\right)\right]} \tag{2}$$

with a reference shear strain $\gamma_r = \tau^{\max}/G_{\max}$ and two constants a and b. $\tau^{\max}$ is the shear strength.

Eq. (1) with its commonly used constants does not consider the strong dependence of the small strain shear modulus of granular soils on the grain size distribution curve. A respective literature review has been given by Wichtmann and Triantafyllidis [8]. For example, Iwasaki and Tatsuoka [5] demonstrated that $G_{\max}$ does not depend on the mean grain size d_{50} but strongly decreases with increasing coefficient of uniformity $C_u = d_{60}/d_{10}$ and with increasing fines content FC. Iwasaki and Tatsuoka [5] performed a single test on each sand. They did not propose an extension of Eq. (1) by the influence of C_u and FC. However, their experiments demonstrated that Hardins equation (1) with its commonly used constants can significantly overestimate the small-strain stiffness of well-graded granular materials. An extension of Eq. (1) considering the influence of the grain size distribution curve is the purpose of the present study.

TESTED MATERIAL, TEST DEVICE, SAMPLE PREPARATION, TESTING PROCEDURE

A natural quartz sand was sieved into 25 single gradations with grain sizes between 0.063 mm and 16 mm. The grains have a subangular shape and the specific weight is $\rho_s = 2.65$ g/cm^3. From these gradations the grain size distribution curves shown in Figure 1 and in the first column of Figures 14 and 15 were mixed. The grain size distribution curves of the materials L1 to L28 (Figure 1) are linear in the semi-logarithmic scale and contain no fines. The eight sands or gravels L1 to L8 (Figure 1a) were used to study the d_{50}-influence. These materials had different mean grain sizes in the range 0.1 mm $\leq d_{50} \leq$ 6 mm and the same coefficient of uniformity C_u = 1.5. The C_u-dependence was examined in tests on the materials shown in Figure 1b. The mean grain sizes of the sands L10 to L26 were d_{50} = 0.2, 0.6 or 2 mm, respectively, while the coefficients of uniformity varied in the range $2 \leq C_u \leq 8$. Two sand-gravel mixtures (L27 and L28, Figure 1c) with larger coefficients of uniformity (C_u = 12.6 or 15.9) were also tested. The influence of the fines content (defined according to German standard as the percentage of grains with diameters $d < 0.063$ mm) was tested by means of the six grain size distribution curves F1 to F6 shown in Figure 1c. These materials have fines contents in the range 0 % $\leq FC \leq$ 20 %. A quartz meal was used for the fines. In the range $d > 0.063$ mm, the grain size distribution curves of the sands F1 to F6 are parallel to those of the materials L1 to L8 (C_u = 1.5).

The resonant column (RC) device used for the present study has been explained in detail by Wichtmann and Triantafyllidis [8]. The device belongs to the "free-free" type, that means both, the top and the base mass are freely rotatable. Cylindrical specimens with a diameter of 10 cm and a height of 20 cm were tested. In order to measure the P-wave velocity the end plates of the RC device have been additionally equipped with a pair of piezoelectric elements. The measuring technique and the analysis of the signals have been presented by Wichtmann and Triantafyllidis [9].

The samples were prepared by air pluviation and tested in the dry condition. For each grain size distribution curve several samples with different initial relative densities $I_{D0} = (e_{\max} -$

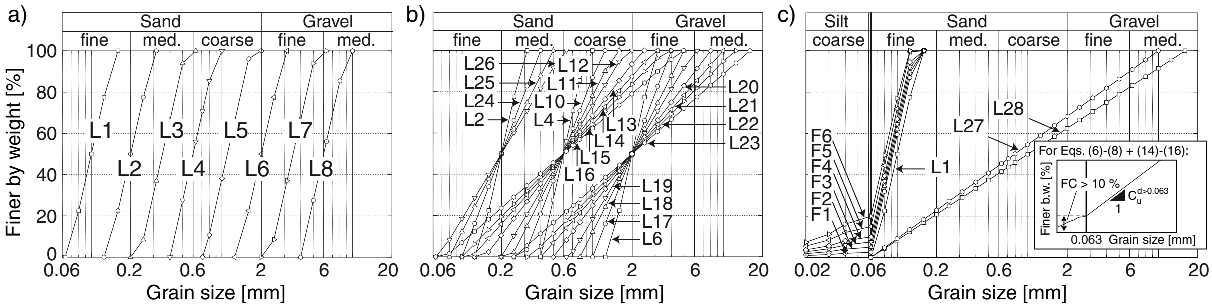

Fig. 1: Tested grain size distribution curves with linear shape

$e)/(e_{\max} - e_{\min})$ were tested. In each test the isotropic stress was increased in seven steps from p = 50 to 400 kPa. At each pressure the small-strain shear modulus $G_{\max}$ and the P-wave velocity v_P were measured. At p = 400 kPa the curves of shear modulus G and damping ratio D versus shear strain amplitude γ were determined. In three additional tests on medium dense samples, the curves $G(\gamma)$ and $D(\gamma)$ were also measured at smaller pressures p = 50, 100 and 200 kPa.

TEST RESULTS AND CORRELATIONS FOR LINEAR GRAIN SIZE DISTRIBUTION CURVES

Influence of d_{50} and C_u on $G_{\max}$

Figure 2 presents results of the RC tests performed on the materials L1 to L8 with C_u = 1.5 and with different mean grain sizes in the range $0.1 \leq d_{50} \leq 6$ mm. Similar as in the tests of Iwasaki and Tatsuoka [5], for constant values of void ratio and mean pressure, no dependence of $G_{\max}$ on d_{50} could be found. The gravel L8 showed slightly lower $G_{\max}$-values which can be explained with an insufficient interlocking between the tested material and the end plates which were glued with coarse sand (Martinez [6]).

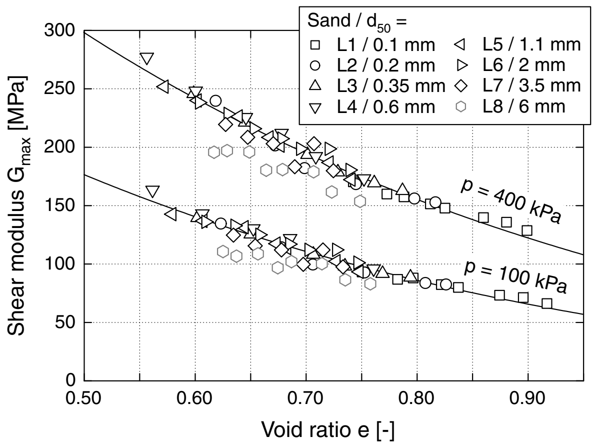

Fig. 2: No dependence of $G_{\max}$ on mean grain size d_{50}, Wichtmann and Triantafyllidis [8]

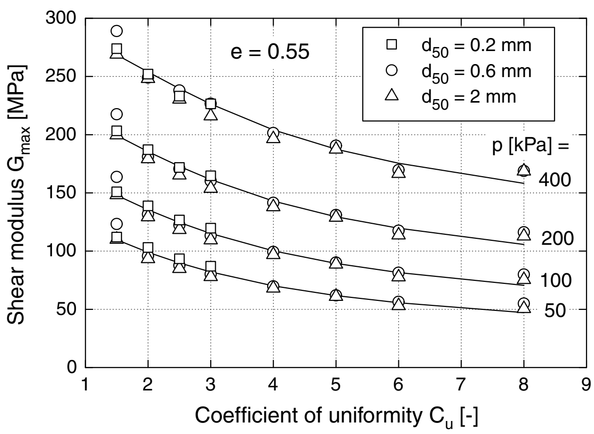

Fig. 3: Decrease of $G_{\max}$ with increasing coefficient of uniformity C_u, data for a constant void ratio e = 0.55, Wichtmann and Triantafyllidis [8]

Figure 3 demonstrates the significant decrease of the small-strain shear modulus $G_{\max}$ with increasing coefficient of uniformity. It was measured in the RC tests on the sands L24 to L26 (d_{50} = 0.2 mm and $2 \leq C_u \leq 3$), L10 to L16 (d_{50} = 0.6 mm and $2 \leq C_u \leq 8$) and L17 to L23 (d_{50} = 2 mm and $2 \leq C_u \leq 8$). For same values of void ratio and pressure, $G_{\max}$ at C_u = 1.5 is approximately twice larger than at C_u = 8. Hardin's equation (1) with its commonly used constants overestimates the $G_{\max}$-values of well-graded granular materials while the shear modulus of uniform sands can be underestimated (Wichtmann and Triantafyllidis [8]).

The parameters A, a and n of Eq. (1) have been obtained from a curve-fitting of Eq. (1) to the $G_{\max}$-data of each sand. Figure 4 shows A, a and n as functions of the coefficient of uniformity C_u. The following correlations could be established (solid lines in Figure 4):

$$a = 1.94 \exp(-0.066 C_u) \tag{3}$$
$$n = 0.40 \, (C_u)^{0.18} \tag{4}$$
$$A = 1563 + 3.13 \, (C_u)^{2.98} \tag{5}$$

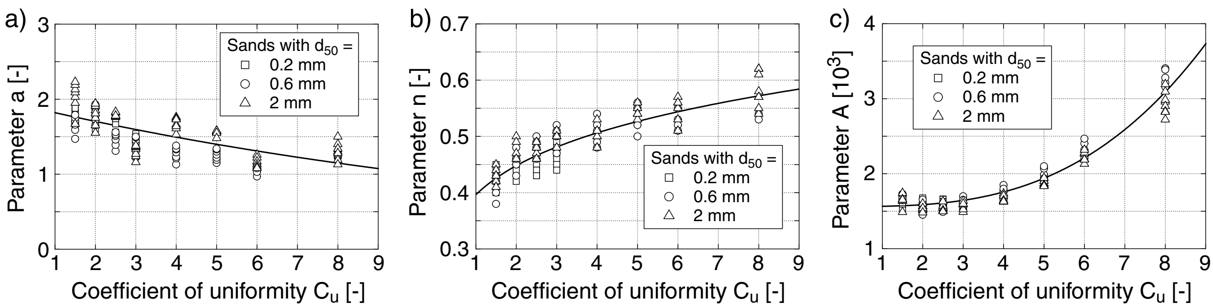

Fig. 4: Correlations of the parameters A, a and n of Eq. (1) with the coefficient of uniformity C_u, Wichtmann and Triantafyllidis [8]

In Figure 5 the shear moduli predicted by Eq. (1) with the correlations (3) to (5) are plotted versus the measured values. All data plot close to the bisecting line (also for the sand-gravel mixtures L27 and L28 with C_u-values up to 16), confirming the good approximation of the experimental data by the extended Hardin's equation. Wichtmann and Triantafyllidis [8] have demonstrated that Eq. (1) with (3) to (5) also predicts well the shear moduli for various sands documented in the literature. Based on the RC test results a correlation of $G_{\max}$ with relative density D_r could be also established (Wichtmann and Triantafyllidis [8]). It is less accurate than Eq. (1) with (3) to (5) but may suffice for practical purposes.

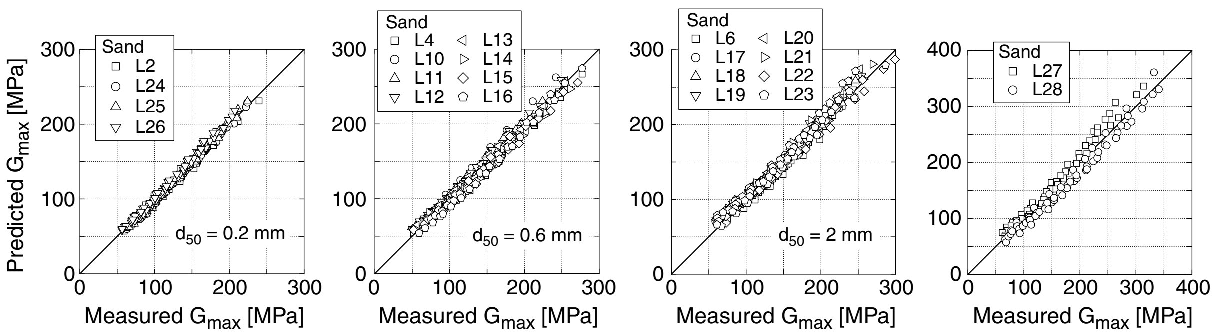

Fig. 5: Comparison of the shear moduli $G_{\max}$ predicted by Eqs. (1) and (3) to (5) with the experimental data, Wichtmann and Triantafyllidis [8]

Influence of fines content on $G_{\max}$

Figure 6 demonstrates the strong decrease of $G_{\max}$ with increasing fines content for $FC \leq 10$ %. This figure shows the data for a constant void ratio $e = 0.825$ obtained from RC tests performed on sands F1 to F6. For the same void ratio and the same pressure, the $G_{\max}$-values for a clean

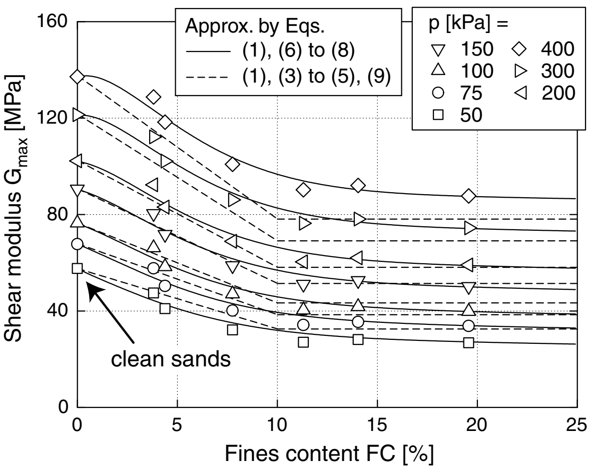

Fig. 6: Decrease of small-strain shear modulus $G_{\max}$ with increasing fines content, data for a constant void ratio $e = 0.825$

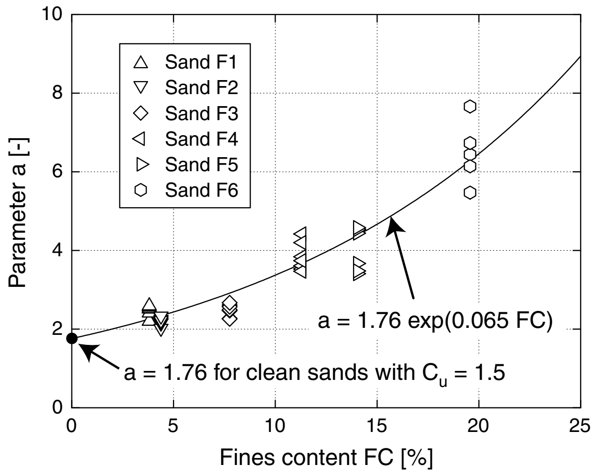

Fig. 7: Parameter a of Eq. (2) as a function of fines content

sand ($FC = 0$) are about twice larger than the shear moduli for a sand with a fines content of 10 %.

The following extension of the correlations (3) to (5) by the influence of the fines content is proposed (see the exemplary plot of a versus FC in Figure 7):

$$a = 1.94 \ \exp(-0.066 C_u) \ \exp(0.065 FC) \tag{6}$$
$$n = 0.40 \ (C_u)^{0.18} [1 + 0.116 \ln{(1 + FC)}] \tag{7}$$
$$A = 0.5[1563 + 3.13 \ (C_u)^{2.98}][\exp(-0.30 FC^{1.10}) + \exp(-0.28 FC^{0.85})] \tag{8}$$

For the parameter A, a very flexible function is necessary. For fines contents $FC > 10$ %, the inclination $C_u^{d>0.063}$ of the grain size distribution curve in the range of grain sizes $d > 0.063$ mm (see the scheme in Figure 1c) should be set into Eqs. (6) to (8) for C_u. The solid curves in Figure 6 were generated with Eqs. (1) and (6) to (8). They confirm the good approximation of the test data by the new correlations. As an alternative, the $G_{\max}$-values obtained for clean sand from Eqs. (1) and (3) to (5) can be reduced by a factor f_r depending on fines content (see the less accurate prediction given as dashed curve in Figure 6):

$$f_r(FC) = \begin{cases} 1 - 0.043 FC & \text{for } FC \leq 10\% \\ 0.57 & \text{for } FC > 10\% \end{cases} \tag{9}$$

Based on the RC test results, for the sands containing fines, $G_{\max}$ could not be correlated with relative density.

Influence of d_{50} and C_u on $M_{\max}$

The constrained elastic modulus $M_{\max} = \rho(v_P)^2$ was calculated from the P-wave velocity. Similar to $G_{\max}$, for constant values of void ratio and pressure, $M_{\max}$ does not depend on the mean grain size (Figure 8) but decreases with increasing coefficient of uniformity (Figure 9).

For each tested material, Eq. (1) with $M_{\max}$ instead of $G_{\max}$ has been fitted to the experimental data:

$$M_{\max} = A \frac{(a - e)^2}{1 + e} (p_{\text{atm}})^{1-n} p^n \tag{10}$$

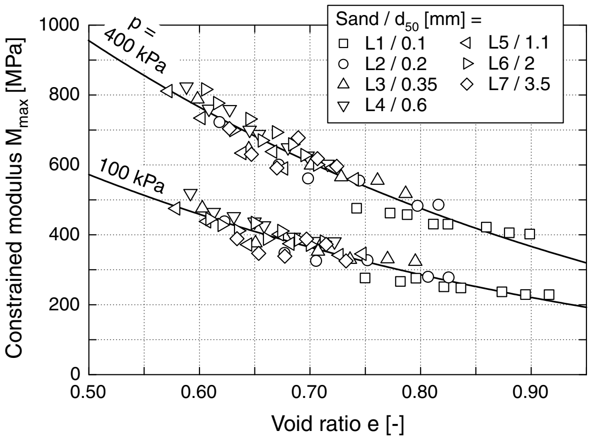

Fig. 8: No influence of mean grain size d_{50} on constrained elastic modulus $M_{\max}$, Wichtmann and Triantafyllidis [9]

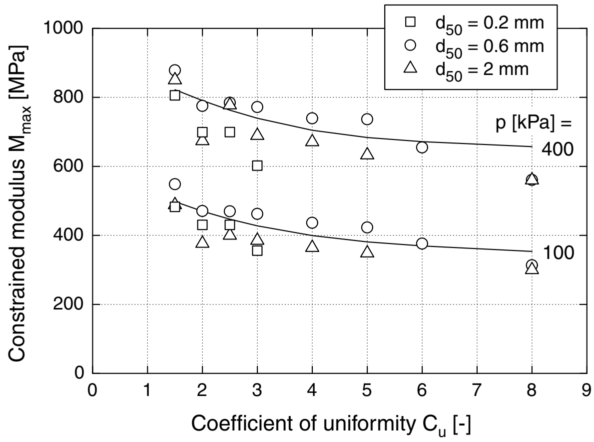

Fig. 9: Decrease of constrained elastic modulus $M_{\max}$ with increasing coefficient of uniformity, Wichtmann and Triantafyllidis [9]

The following correlations between the parameters A, a and n of Eq. (10) and C_u could be derived:

$$a = 2.16 \exp(-0.055\ C_u) \tag{11}$$
$$n = 0.344(C_u)^{0.126} \tag{12}$$
$$A = 3655 + 26.7(C_u)^{2.42} \tag{13}$$

The $M_{\max}$-values calculated from Eqs. (10) to (13) agree well with the experimental data (Wichtmann and Triantafyllidis [9]). The correlation between $M_{\max}$ and relative density is rather rough. With $G_{\max}$ from Eqs. (1) and (3) to (5) and $M_{\max}$ from Eqs. (10) to (13), Poisson's ratio ν can be calculated. For constant values of void ratio and pressure, ν increases with increasing C_u (Wichtmann and Triantafyllidis [9]).

Influence of fines content on $M_{\max}$

Similar to $G_{\max}$, the constrained elastic modulus $M_{\max}$ decreases with increasing fines content, at least in the range $FC \leq 10$ % (Figure 10). Therefore, the correlations (11) to (13) have been extended by the influence of FC:

$$a = 2.16\ \exp(-0.055\ C_u)(1 + 0.116 FC) \tag{14}$$
$$n = 0.344(C_u)^{0.126}[1 + 0.125 \ln(1 + FC)] \tag{15}$$
$$A = 0.5[3655 + 26.7(C_u)^{2.42}][\exp(-0.42 FC^{1.10}) + \exp(-0.52 FC^{0.60})] \tag{16}$$

The solid curves in Figure 10 were generated with Eqs. (10) and (14) to (16). They approximate the experimental data well. For a simplified procedure, the $M_{\max}$-value obtained for clean sand from Eqs. (10) to (13) can be reduced by a factor f_r (see the prediction given as dashed curve in Figure 10):

$$f_r(FC) = \begin{cases} 1 - 0.041 FC & \text{for } FC \leq 10\% \\ 0.59 & \text{for } FC > 10\% \end{cases} \tag{17}$$

The small dependence of Poisson's ratio ν on fines content can be neglected for practical purposes.

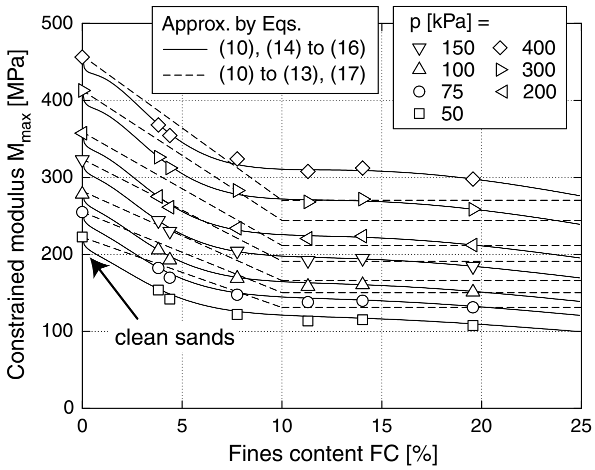

Fig. 10: Decrease of small-strain constrained elastic modulus $M_{\max}$ with increasing fines content, data for a constant void ratio $e = 0.825$

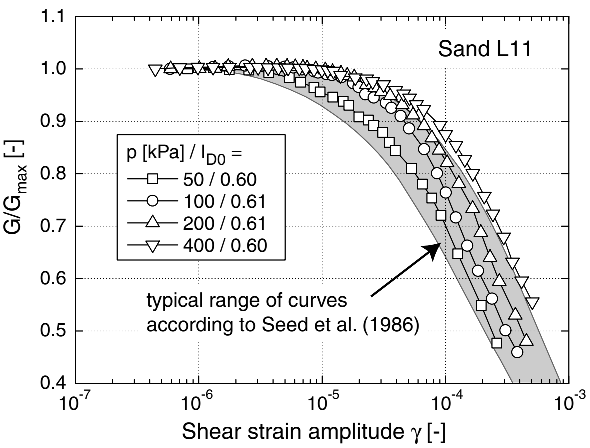

Fig. 11: Typical curves $G(\gamma)/G_{\max}$ for four different pressures, shown exemplary for sand L11

Influence of d_{50} and C_u on the curves $G(\gamma)/G_{\max}$ and $D(\gamma)$

Figure 11 presents typical curves $G(\gamma)/G_{\max}$ measured in four tests with different pressures. In accordance with the literature, the modulus degradation is larger for smaller pressures. The curves $G(\gamma)/G_{\max}$ measured in the present study fall in the range of typical values specified by Seed et al. [7] (gray region in Figure 11). An influence of density on the curves $G(\gamma)/G_{\max}$ could not be detected.

The modulus degradation becomes larger with increasing coefficient of uniformity. This is evident in Figure 12: For a certain value of shear strain amplitude γ, the ratio $G/G_{\max}$ decreases with increasing C_u. No significant influence of the mean grain size on the curves $G(\gamma)/G_{\max}$ could be found. For each material, $G/G_{\max}$ was plotted versus the normalized shear strain amplitude γ/γ_r and Eq. (2) was fitted to the data. The reference shear strain γ_r was determined from monotonic triaxial tests. The parameter b in Eq. (2) was set to 1 which is sufficient in order to describe the modulus degradation curves (see also Hardin and Kalinski [3]). The relationship between the parameter a in Eq. (2) and the coefficient of uniformity C_u (Figure 13) can be described by:

$$a \quad = \quad 1.070 \ln (C_u) \tag{18}$$

Damping ratio D increases with decreasing pressure, but does not depend on density. A comparison of the damping ratios measured for the sands L1 to L8 revealed that D does not significantly depend on mean grain size. The influence of the coefficient of uniformity on D depends on shear strain amplitude and pressure. For larger pressures, D increases with C_u, independently of the shear strain amplitude. For smaller pressures, D is almost independent of C_u if the shear strain amplitudes is small, while a decrease of D with C_u was observed at larger γ-values.

During the increase of shear strain amplitude the axial deformation of the samples was measured. From this data the threshold shear strain amplitude γ_{tv} at the onset of settlement could be determined. The threshold shear strain amplitude γ_{tl} at the transition from the linear to the nonlinear elastic behavior was defined as the amplitude for which the shear modulus has decreased to 99 % of its initial value, i.e. $G = 0.99 G_{\max}$. The threshold amplitudes γ_{tl} and γ_{tv} neither depend on d_{50} nor on C_u.

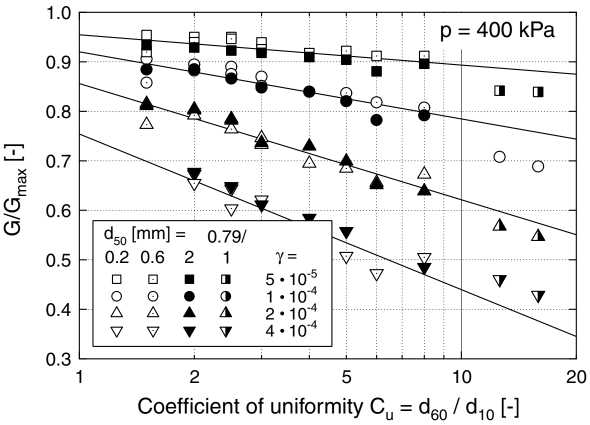

Fig. 12: Factor $G/G_{\max}$ as a function of C_u, plotted for different shear strain amplitudes

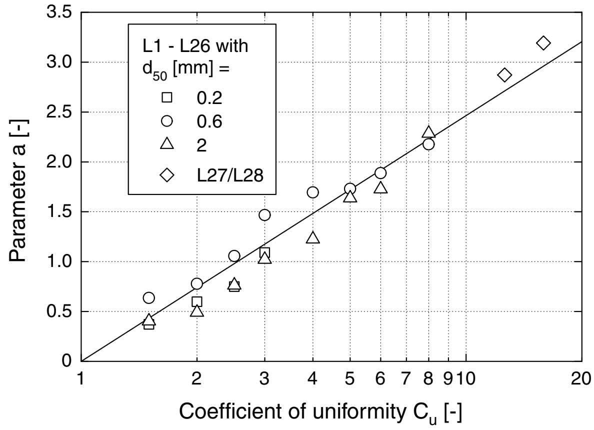

Fig. 13: Correlation of the parameter a in Eq. (2) with the coefficient of uniformity

Influence of fines content on the curves $G(\gamma)/G_{\max}$ and $D(\gamma)$

Hardly no influence of the fines content on the curves $G(\gamma)/G_{\max}$ and $D(\gamma)$ could be found in the RC tests on sands F1 to F6. However, due to the decrease of $G_{\max}$ with increasing FC, the reference shear strain $\gamma_r = \tau^{\max}/G_{\max}$ significantly increases with increasing fines content, resulting in an increase of the parameter a in Eq. (2). Therefore, the following extension of Eq. (18) is proposed:

$$a \quad = \quad 1.070 \ln{(C_u)} \;\; \exp(0.053FC) \qquad (19)$$

For small pressures ($p = 50$ kPa) the damping ratio D decreases by almost a factor 4 when the fines content is increased from 0 to 10 %. For larger fines contents the damping ratio stays almost constant. For larger pressures ($p = 400$ kPa) the decrease of D with FC is less pronounced.

While the linear elastic threshold shear strain amplitude γ_{tl} does hardly depend on fines content, the cumulative threshold shear strain amplitude γ_{tv} increases with increasing FC.

APPLICABILITY OF THE CORRELATIONS FOR PIECEWISE LINEAR, GAP-GRADED, S-SHAPED AND OTHER SMOOTHLY SHAPED GRAIN SIZE DISTRIBUTION CURVES

All correlations presented above have been developed based on experimental data for grain size distribution curves with a linear shape in the semi-logarithmic scale. In the meantime, the new correlations have been inspected for piecewise linear, gap-graded, S-shaped and other smoothly shaped grain size distribution curves. All these materials did not contain fines. Figures 14 and 15 show some of the tested grain size distribution curves, together with the measured $G_{\max}(e)$- and $M_{\max}(e)$-data for pressures $p = 100$ and 400 kPa. In Figures 14 and 15, the shear moduli $G_{\max}(e)$ predicted by Eq. (1) with (3) to (5) and the constrained elastic moduli $M_{\max}(e)$ predicted by Eqs. (10) to (13) have been added as thick solid curves. These curves were generated using $C_u = d_{60}/d_{10}$ as input for the correlations. The equivalent linear grain size distribution curves, which have the same d_{10}- and C_u-values, are shown as thick solid lines in the first column of diagrams in Figures 14 and 15. For most of the "more complicated" grain size distribution curves, the experimental data is well approximated by the new correlations. However, for a few materials (see e.g. PL7 and GG2 in Figures 14 and 15) too low $G_{\max}$- and $M_{\max}$-values are predicted. Therefore, a possible improvement of the prediction by using the equivalent linear grain size distribution curves shown as dashed thick lines in the first column of diagrams in Figures 14 and 15 has been checked. They have the same d_{10} but the inclination $C_{u,A}$ is chosen

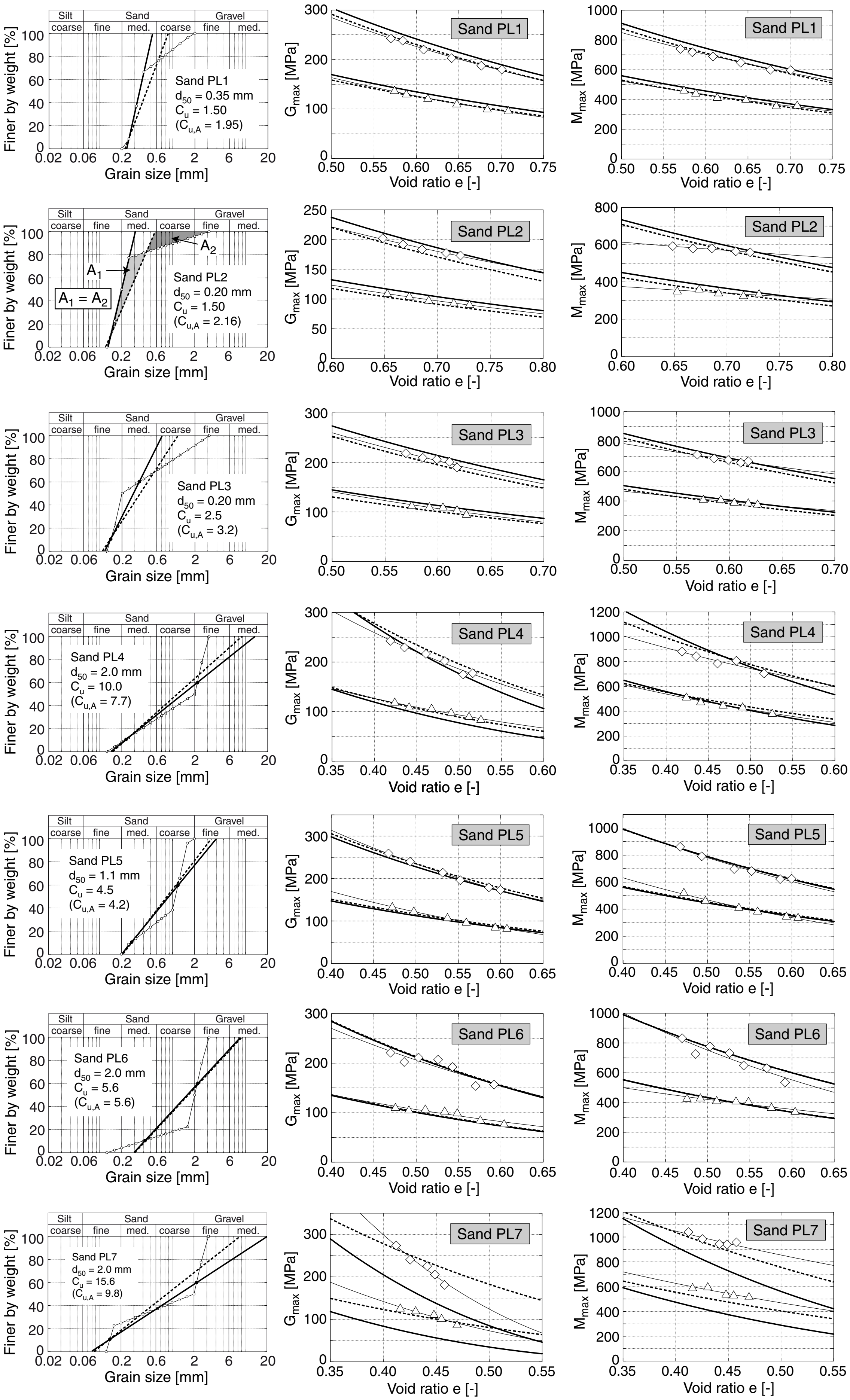

Fig. 14: Results from tests on piecewise linear grain size distribution curves

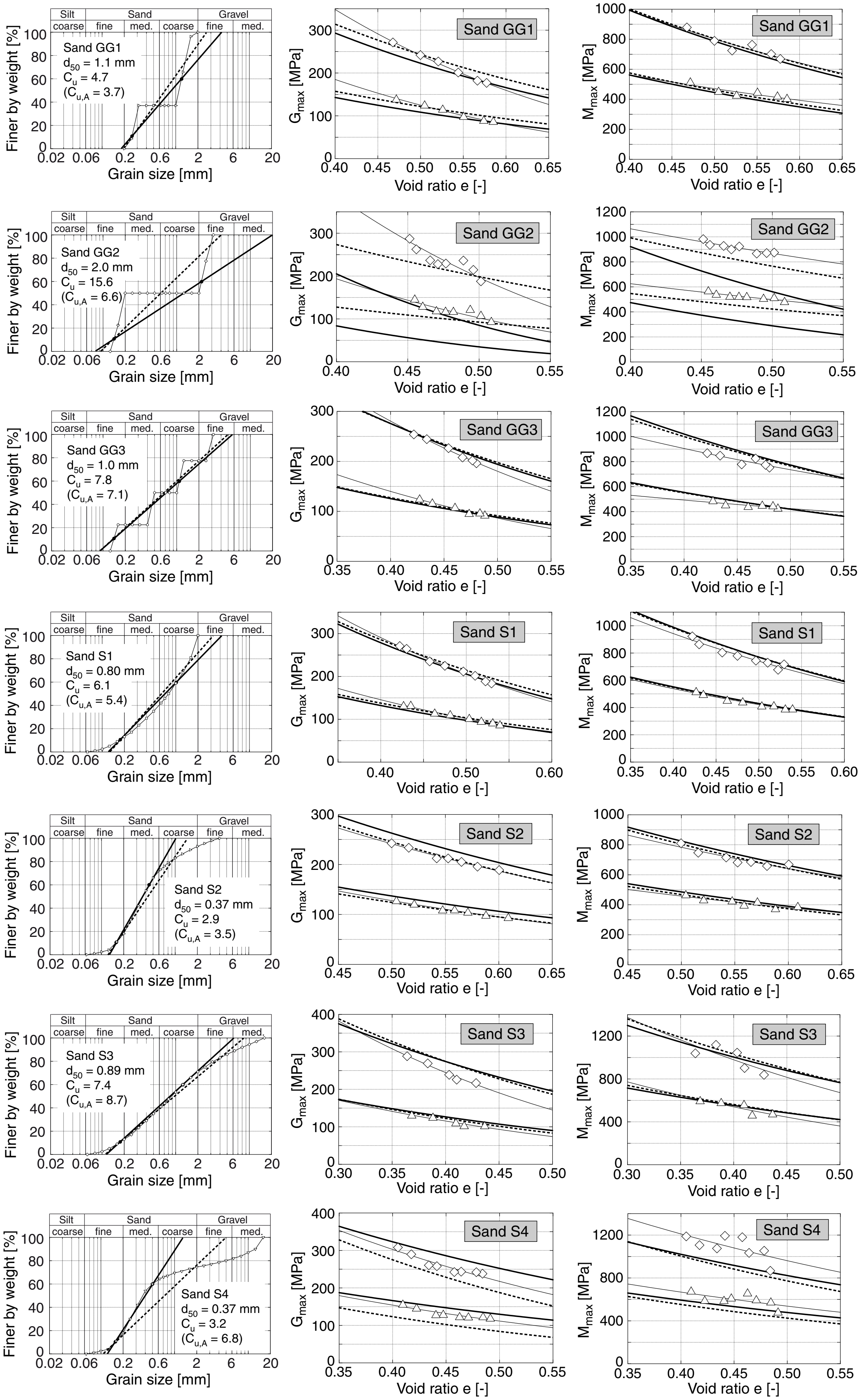

Fig. 15: Results from tests on gap-graded and smoothly shaped grain size distribution curves

such way that the areas enclosed between the original and the equivalent linear curve, above and below the original curve, are equal (see the scheme in the diagram for sand PL2, Figure 14). For most tested materials, in particular for PL7 and GG2, the difference between the measured and the predicted $G_{\max}$- and $M_{\max}$-values is less if $C_{u,A}$ is used instead of $C_u = d_{60}/d_{10}$. It can be concluded that the new correlations work well also for "more complicated" grain size distribution curves. It is recommended to apply the correlations with $C_{u,A}$ instead of C_u.

SUMMARY AND CONCLUSIONS

Approx. 650 resonant column tests with additional P-wave measurements have been performed on 64 quartz sands with different grain size distribution curves. First, grain size distribution curves with a linear shape in the semi-logarithmic scale were tested. These tests showed that the small-strain shear modulus $G_{\max}$ and the small-strain constrained elastic modulus $M_{\max}$ are independent of mean grain size d_{50} but strongly decrease with increasing coefficient of uniformity $C_u = d_{60}/d_{10}$. A fines content leads to a further reduction of $G_{\max}$ and $M_{\max}$. The well-known Hardin's equation for $G_{\max}$ has been extended by the influence of the grain size distribution curve. For that purpose the parameters have been correlated with C_u and FC. A similar set of equations has been developed for $M_{\max}$.

For a certain shear strain amplitude, the modulus degradation factor $G(\gamma)/G_{\max}$ decreases with increasing coefficient of uniformity, but hardly depends on fines content. An empirical formula for the modulus degradation factor has been extended by the influence of the grain size distribution curve. Damping ratio D decreases or increases with C_u, depending on pressure and shear strain amplitude. A fines content reduces the damping ratio. The decrease is more pronounced at low pressures. The linear elastic threshold shear strain amplitude γ_{tl} depends neither on C_u nor on FC. The cumulative threshold shear strain amplitude γ_{tv} is not affected by the coefficient of uniformity, but increases with increasing fines content.

In a second test series, piecewise linear, gap-graded, S-shaped and other smoothly shaped grain size distribution curves have been tested. For most of these materials, the $G_{\max}$ and $M_{\max}$-values predicted by Hardin's equation with the new correlations agree well with the measurements. For a few materials, a better congruence between the predicted and the experimental data is achieved when the new correlations are applied with an inclination factor $C_{u,A}$ instead of $C_u = d_{60}/d_{10}$. $C_{u,A}$ is defined as the uniformity coefficient of an equivalent linear grain size distribution curve, for which the areas enclosed between the original and the equivalent linear curve, above and below the original curve, are equal.

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