Small-strain constrained elastic modulus of clean quartz sand with various grain size distribution

T. Wichtmanni) Th. Triantafyllidisii)

Abstract: Approx. 120 resonant column (RC) tests with additional P-wave velocity measurements using piezoelectric elements have been performed on 19 clean quartz sands with piecewise linear, gap-graded, S-shaped or other smoothly shaped grain size distribution curves. For each material different pressures and densities were tested. It is demonstrated that the extended empirical equations for the small-strain constrained elastic modulus proposed by the authors in an earlier paper work well also for most of the more complex grain size distribution curves tested in the present study. These equations considering the influence of the uniformity coefficient of the grain size distribution curve were developed based on data from tests on linear gradations. A further improvement of the prediction for the more complex grain size distributions can be achieved if the correlation equations are applied with a specially defined average inclination of the grain size distribution curve. Such improvement is demonstrated not only for the small-strain constrained elastic modulus, but also for small-strain shear modulus, modulus degradation and Poisson’s ratio.

Keywords: P-wave velocity; small strain constrained elastic modulus; quartz sand; grain size distribution curve; uniformity coefficient

1 Introduction

Similar to the small strain shear modulus \(G_{\text{max}}\) (Iwasaki & Tatsuoka [8], Wichtmann & Triantafyllidis [9]), the small-strain constrained elastic modulus \(M_{\text{max}}\) of clean quartz sand is strongly dependent on the uniformity coefficient \(C_u = d_{50}/d_{10}\) of the grain size distribution curve (Wichtmann & Triantafyllidis [10]). For constant values of void ratio and pressure, \(G_{\text{max}}\) and \(M_{\text{max}}\) decrease with increasing \(C_u\), while they are rather independent of mean grain size \(d_{50}\). The common empirical formulas for the small-strain stiffness were developed based on tests on rather uniform sands. It has been demonstrated that these equations should not be applied to well-graded granular materials since they may strongly overestimate the stiffness of well-graded soils [9, 10].

In order to consider the influence of the uniformity coefficient in an empirical formula for \(M_{\text{max}}\), the authors [10] have proposed the following set of equations, based on the well-known empirical formula of Hardin [5, 7]:

\[
M_{\text{max}} = A \left(\frac{a - e}{1 + e}\right)^2 \left(\frac{p}{p_{\text{atm}}}\right)^n p_{\text{atm}}
\]

\(a = 2.16 \exp(-0.055 C_u)\)

\(n = 0.344 C_u^{0.126}\)

\(A = 3655 + 26.7 C_u^{2.42}\)

with void ratio \(e\), mean effective confining pressure \(p\) and atmospheric pressure \(p_{\text{atm}} = 100\) kPa. The correlations (2) to (4) are based on 163 resonant column (RC) tests with

P-wave measurements performed on 25 grain size distribution curves with linear shape in a diagram with semilogarithmic scale. The testing methodology was the same as that applied in the present study (see next section). As an alternative to Eqs. (1) to (4), a relationship between \(M_{\text{max}}\) and relative density \(D_r\) has been established in [10]:

\[
M_{\text{max}} = 2316 \left(1 + 1.07 D_r[\%]/100\right) p_{\text{atm}}^{1-0.39} p^{0.39}
\]

with \(D_r = (e_{\text{max}} - e)/(e_{\text{max}} - e_{\text{min}})\) calculated with the minimum and maximum void ratios \(e_{\text{min}}\) and \(e_{\text{max}}\) from standard tests (determined according to DIN 18126 in case of the present study).

The present paper investigates whether the extended empirical equations (1) to (5) can be also applied to more complex grain size distribution curves. For this purpose experimental data collected for piecewise linear, gap-graded, S-shaped and other smoothly shaped grain size distribution curves are analyzed.

2 Tested material, test device and testing procedure

The specially composed grain size distribution curves tested in the present study are collected in Figure 1. They are also shown separately in the first and third column of diagrams in Figures 5 and 6. The original material is a natural quartz sand with subangular grains originating from a sand pit near Dorsten, Germany. Some of the grain size distribution curves (materials PL1 - PL7) have a piecewise linear shape with varying inclinations and inflection points. Others are gap-graded (materials GG1 - GG8) with varying span of missing grain sizes. S-shaped and other smoothly shaped grain size distribution curves (materials S2 - S6) were also tested. The values of mean grain size \(d_{50}\), uniformity coefficient \(C_u\) and curvature index \(C_c = (d_{50})^2/(d_{10} d_{90})\) of the tested grain size distribution curves are given in Table...
Fig. 1: Specially composed grain size distribution curves tested in the present study

![Grain size distribution curves](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>$d_{50}$ [mm]</th>
<th>$C_u$ [-]</th>
<th>$C_{u,A}$ [-]</th>
<th>$C_c$ [-]</th>
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<tbody>
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</tr>
<tr>
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<td>5.6</td>
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</tr>
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<td>3.7</td>
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Table 1: Mean grain size $d_{50}$, uniformity coefficient $C_u$, average inclination $C_{u,A}$ and curvature index $C_c$ of the tested grain size distribution curves

1. $M_{\text{max}}$ data was not available for the materials GG7 and S5, for which $G_{\text{max}}$ data has been analyzed by the authors in [12].

A scheme of the resonant column (RC) device used for the present study is shown in Fig. 2. It is of the “free - free” type, meaning both the top and the base mass are freely rotatable. The cuboidal top mass is equipped with two electrodynamic exciters each accelerating a small mass. This acceleration and the resulting acceleration of the top mass are measured with acceleration transducers. From these signals the torsional moment $M(t)$ and the angle of twist $\theta(t)$ at the top of the sample can be calculated. The sample is enclosed in a pressure cell. The state of stress is almost isotropic. A small stress anisotropy results from the weight of the top mass ($m \approx 9$ kg), such that the vertical stress $\sigma_1$ is slightly higher than the lateral one $\sigma_2$. However, for higher cell pressures this anisotropy is of secondary importance. Furthermore, test results of Yu & Richart [14] reveal that a stress anisotropy becomes significant only near failure.

A sinusoidal electrical signal is generated by a function generator, amplified and applied to the electrodynamic exciters. The frequency of excitation is varied until the resonant frequency $f_R$ of the system composed of the two end masses and the specimen has been found. By definition, this is the case when $M(t)$ and $\theta(t)$ have a phase-shift of $\pi/2$ in time $t$. The secant shear modulus

$$G = \left(\frac{2\pi h f_R}{a}\right)^2 \rho$$

is calculated from the resonant frequency, the height $h$ and the density $\rho$ of the specimen. The parameter $a$ is obtained from Eq. (7):

$$a \tan(a) - \frac{J^2}{J_0 J_L} \tan(a) = \frac{J}{J_0} + \frac{J}{J_L}$$

In Eq. (7) $J$ is the polar mass moment of inertia of the specimen and $J_0 = 1.176$ kg m$^2$ and $J_L = 0.0663$ kg m$^2$ are the respective values of the base mass and the top mass (Fig. 2a).

Different shear strain amplitudes can be tested by varying the amplitude of the torsional excitation. All tested specimens had a full cross section and measured $d = 10$ cm in diameter and $h = 20$ cm in height. The variation of the shear strain amplitude with radius $r$ is considered by calculating a mean value over the sample volume:

$$\overline{\gamma} = \frac{1}{V} \int_V \gamma(r, x) \, dV$$

This mean value is simply denoted by $\gamma$ in the analysis of the test results in this paper. The shear strain amplitudes
**Fig. 4:** Small-strain constrained elastic modulus $M_{\text{max}}$ as a function of void ratio for different confining pressures (upper row of diagrams) and as a function of confining pressure for different initial relative densities (lower row of diagrams). The data is shown for a piecewise linear (PL1), a gap-graded (GG2) and an S-shaped (S1) grain size distribution curve.

**Fig. 3:** Example of transmitted and received signals, interpretation of travel time $t_t$, from [10]

that can be tested in the RC device lie in the range $5 \times 10^{-7} \leq \gamma \leq 5 \times 10^{-4}$.

For P-wave measurements the specimen end plates have been additionally equipped with piezoelectric elements. The transducers are similar to those explained by Brignoli et al. [4]. A single sinusoidal signal with a frequency of $f = 20$ kHz was applied to the element in the base pedestal. The travel time $t_t$ has been determined from the first arrival of the signal received at the top cap. Typical signals are presented in Figure 3.

Delay times in cables, amplifiers, etc. have been subtracted from $t_t$. Based on the literature the strain amplitudes generated in the soil using this type of P-wave sensors are assumed to be less than $10^{-6}$. The constrained elastic modulus is calculated from the P-wave velocity using $M_{\text{max}} = q(v_P)^2$ with soil density $q$. In [13] it has been demonstrated that the $G_{\text{max}}$-values obtained from S-wave velocity measurements by means of piezoelectric elements are close to the $G_{\text{max}}$-values measured with the RC device. Therefore, the $G_{\text{max}}$ values obtained with the RC function of the test device and the $M_{\text{max}}$ values derived from the P-wave measurements can be directly compared in order to calculate Poisson’s ratio.

The lateral deformations and the settlement of the samples due to the increase of confining pressure or the application of shear strain cycles with higher amplitude $\gamma$ were measured with non-contact displacement transducers.

All specimens were prepared by air pluviation and tested in the air-dry condition. For each material several specimens with different initial relative densities $D_{\text{so}}$ were tested. The mean effective confining pressure $p$ was increased step-wise from $p = 50$ to $400$ kPa. At each pressure $p$ the small strain shear modulus $G_{\text{max}}$ and the P-wave velocity $v_P$ were measured after a resting period of 5 minutes, in order to obtain a similar "aging" (Afifi & Woods [2], Afifi & Richart [1], Baxter [3]) of the samples. Finally, the curves $G(\gamma)$ and $D(\gamma)$ were measured at $p = 400$ kPa. In three additional tests on medium dense specimens the modulus degradation and the damping ratio were also measured at $p = 50$, 100 and 200 kPa for each material.

### 3 Test results for small-strain constrained elastic modulus

Typical test results for a piecewise linear (PL1), a gap-graded (GG2) and an S-shaped (S1) grain size distribution curve are shown in Figure 4. The upper row of diagrams in Figure 4 gives the small-strain constrained elastic modulus $M_{\text{max}}$ as a function of void ratio for different confining pres-
Fig. 5: Small-strain constrained elastic modulus $M_{\max}(e)$ for piecewise linear and gap-graded grain size distribution curves: Comparison of test data for pressures $p = 100$ and $400$ kPa with data predicted by Eqs. (1) to (4) using either $C_u$ (thick solid curves) or $C_{u,A}$ (dashed curves) as input for the correlations. The thin solid curves are the best-fit curves of Eq. (1) for the data at pressure $p = 100$ or $400$ kPa, respectively.
Fig. 6: Small-strain constrained elastic modulus $M_{\text{max}}(e)$ for smoothly shaped grain size distribution curves: Comparison of test data for pressures $p = 100$ and 400 kPa with data predicted by Eqs. (1) to (4) using either $C_u$ (thick solid curves) or $C_{u,A}$ (dashed curves) as input for the correlations. The thin solid curves are the best-fit curves of Eq. (1) for the data at pressure $p = 100$ or 400 kPa, respectively.

Fig. 7: Small strain constrained elastic modulus $M_{\text{max}}$ for a void ratio $e = 0.55$ and pressures $p = 100$ and 400 kPa as a function of uniformity coefficient $C_u$; Comparison of data from the present test series (filled circles) with data measured for linear grain size distribution curves (open symbols, [10]).

Tests on linear grain size distribution curves (Wichtmann & Triantafyllidis, 2010):
- $d_{50} = 0.2$ mm
- $d_{50} = 0.6$ mm
- Present test series

The small-strain constrained elastic modulus $M_{\text{max}}(e)$ predicted by Eqs. (1) to (4) for $p = 100$ and 400 kPa have been added as thick solid curves in the second and fourth column of diagrams in Figures 5 and 6. These curves were generated using $C_u$ as input for the correlations (2) to (4). The equivalent linear grain size distribution curves, which have the same $d_{50}$- and $C_{u}$-values as the tested grain size distribution curves, are shown as thick solid lines in the first and third column of diagrams in Figures 5 and 6. For most of the more complex grain size distribution curves, the experimental data is well approximated by Eqs. (1) to (4). The measured moduli of some materials (e.g., PL1, PL2, PL4, GG2, GG4, GG8, S4 and S6) are slightly overestimated by Eqs. (1) to (4) while the moduli of other materials (e.g., S3) are slightly underestimated. A relatively large un-
derestimation of the experimental $M_{\text{max}}$ data (up to factor 1.7) was observed for the two materials PL7 and GG6.

An improvement of the $M_{\text{max}}$ prediction, in particular for the materials PL7 and GG6, can be achieved if the correlations (2) to (4) are applied with an average inclination $C_{u,A}$ of the grain size distribution curve. The definition of $C_{u,A}$ is explained in Figure 9, using the grain size distribution curve of sand PL2 as an example. While the equivalent linear gradation has the same $d_{10}$ value as the tested grain size distribution curve, its inclination $C_{u,A}$ is chosen such that the areas enclosed between the tested and the equivalent linear curve, above and below the tested curve, are equal in the semi-logarithmic scale ($A_1 = A_2$ in Figure 9). The equivalent linear gradations with inclinations $C_{u,A}$ are shown as dashed thick lines in the first and third column of diagrams in Figures 5 and 6. The $C_{u,A}$-values are collected in Table 1 and are also specified in Figures 5 and 6. For most tested materials, in particular for PL7 and GG6, the differences between the measured and the predicted $M_{\text{max}}$-values are less if $C_{u,A}$ is used instead of $C_u$. This becomes also clear from Table 2 where the percentage of predicted $M_{\text{max}}$ data differing either $\leq 10\%$ or $\leq 20\%$ from the measured $M_{\text{max}}$ data is provided. For comparison, in Table 2 similar data is given for the linear grain size distribution curves tested by the authors [10].

Another proof for the good prediction quality of the extended empirical equations is provided in Figure 8. Each diagram in Figure 8 collects the $M_{\text{max}}(\varepsilon)$ data at $p = 400$ kPa for certain $C_{u,A}$ values, measured for the linear or the more complex grain size distribution curves (for the linear curves $C_u = C_{u,A}$ holds). The experimental data for both types of gradations is well approximated by the curves $M_{\text{max}}(\varepsilon)$.
predicted by Eqs. (1) to (4) applied with $C_{u,A}$ (solid curves in Figure 8).

In agreement with the results for linear grain size distribution curves [10], the values in the last two columns of Table 2 demonstrate that the prediction by Eq. (5) using relative density $D_r$ as input is much less accurate than that of Eqs. (1) to (4) with either $C_u$ or $C_{u,A}$.

4 Reanalysis of small-strain shear modulus and modulus degradation data in terms of average inclination of grain size distribution curve

In [12] the authors presented the data of small-strain shear modulus $G_{\text{max}}$ and modulus degradation ratio $G(\gamma)/G_{\text{max}}$ measured for the grain size distribution curves shown in Figure 1. Since an improved $M_{\text{max}}$ prediction was found for the more complex grain size distribution curves if the correlations (2) to (4) were applied with the average inclination $C_{u,A}$ instead of the conventional $C_u$, a possible improvement by using $C_{u,A}$ was also inspected for $G_{\text{max}}$ and $G(\gamma)/G_{\text{max}}$, re-analyzing the data from [12]. Figure 10 presents the experimental $G_{\text{max}}(e)$ data at pressures $p = 100$ and 400 kPa for some of the tested sands. The prediction by the equation (Hardin [5, 7])

$$G_{\text{max}} = A \left( \frac{a - e}{1 + e} \right)^2 \left( \frac{p}{p_{\text{atm}}} \right)^n p_{\text{atm}}$$

with the correlations (10) to (12) proposed by the authors in [9]

$$n = 0.40 C_u^{0.18}$$

$$A = 1563 + 3.13 C_u^{2.98}$$

with $a = 1.94 \exp(-0.066 C_u)$ (11) (12)

has been added as thick solid curves in Figure 10. Eqs. (10) to (12) have been evaluated with $C_u$ for generating these curves. The prediction by Eqs. (9) to (12) using $C_{u,A}$ instead of $C_u$ is provided as thick dashed curves in Figure 10. For most of the tested materials, the prediction with $C_{u,A}$ lies closer to the experimental data than the curves generated with $C_u$. In particular, an improvement of the $G_{\text{max}}$ prediction can be achieved for sands PL7, GGS and S5, which either contain a large amount of gravel (about 60% in the case of S5) or are primarily composed of two components with significantly different grain size (fine sand and fine gravel in case of PL7 and GGS).

A quantitative analysis of the differences between measured and predicted $G_{\text{max}}$ data is provided in Table 3, where the percentage values of predicted $G_{\text{max}}$ data differing either $\leq 10\%$ or $\leq 20\%$ from the measured data are given. The differences between measured and predicted data are less if $C_{u,A}$ is applied in the prediction instead of $C_u$. For comparison, the values for the linear grain size distribution curves tested by the authors [9] are also provided in Table 3.

Another proof for the good prediction quality of Eqs. (9) to (12) with the average inclination $C_{u,A}$ is provided in Figure 11 where the $G_{\text{max}}(e)$ data for $p = 400$ kPa is presented in a similar manner as the $M_{\text{max}}(e)$ data in Figure 8. Each diagram in Figure 11 collects the shear moduli measured for sands having a similar $C_{u,A}$-value. For $C_{u,A} \approx \text{constant},$
the $G_{\text{max}}(e)$ data of the various tested sands is similar - irrespectively if the grain size distribution curve is linear or more complex. The experimental data is well approximated by Eqs. (9) to (12) if the correlations are applied with $C_{u,A}$ (see the solid curves in Figure 11).

The modulus degradation curves $G(\gamma)/G_{\text{max}}$ measured for some of the tested materials at pressures $p = 50$ and 400 kPa are shown in Figure 12. The prediction by the empirical formula proposed by Hardin & Drnevich [6]

$$
\frac{G}{G_{\text{max}}} = \frac{1}{1 + \frac{\gamma}{\gamma_r} [1 + a \exp (-\frac{\gamma}{\gamma_r})]}
$$

(13)

with the correlation developed by the authors in [11]

$$
a = 1.070 \ln(C_u)
$$

(14)

has been added as thick solid curves in Figure 12, using $C_u$ as input for the correlation. The reference shear strain $\gamma_r = \tau_{\text{max}}/G_{\text{max}}$ has been evaluated using $\tau_{\text{max}} = p \sin \varphi_P$ for isotropic stress conditions, applying the following correlation for the peak friction angle $\varphi_P$ proposed in [11]:

$$
\varphi_P = 34.0^o \exp(0.27 (D_r [/%])/100)^{1.3}
$$

(15)

If Eq. (14) is evaluated with $C_{u,A}$ instead of $C_u$, the thick dashed curves shown in Figure 12 are obtained. For most of the tested materials, the differences between the $G(\gamma)/G_{\text{max}}$ predictions using either $C_u$ or $C_{u,A}$ are small. However, for the materials PL7, GG6 and S5 a better approximation of the experimental data can be achieved if the correlation (14) is applied with $C_{u,A}$ as input. The slightly better approximation of the experimental data with $C_{u,A}$ instead of $C_u$ becomes clear also from Table 4 where the percentage values of predicted $G(\gamma)/G_{\text{max}}$ data differing either $\leq 0.05$ or $\leq 0.1$ from measured $G_{\text{max}}$ data. Only data with $G/G_{\text{max}} < 0.9$ is considered. These values are mean values over all tested materials with either linear (second row) or more complex (first row) grain size distribution.

Table 3: Percentage values of $G_{\text{max}}$ data differing either $\leq 10\%$ or $\leq 20\%$ from the experimental data. These values are mean values over all tested materials with either linear (second row) or more complex (first row) grain size distribution.

<table>
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Table 4: Percentage values of predicted $G_{\text{max}}$ data differing $\leq 0.05$ or $\leq 0.1$ from measured $G_{\text{max}}$ data. Only data with $G_{\text{max}}/G_{\text{max}} < 0.9$ is considered. These values are mean values over all tested materials with either linear (second row) or more complex (first row) grain size distribution.

<table>
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<th>$G_{\text{max}}/G_{\text{max}} &lt; 0.9$</th>
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</table>
from tests on linear gradations are applied with an average inclination \( C_{u,A} \) of the grain size distribution curve instead of \( C_u \).

5 Poisson’s ratio

Poisson’s ratio \( \nu \) can be calculated from

\[
\nu = \frac{\alpha}{4(1-\alpha)} + \sqrt{\left( \frac{\alpha}{4(1-\alpha)} \right)^2 - \frac{\alpha - 2}{2(1-\alpha)}} \tag{16}
\]

with \( \alpha = \frac{M_{\text{max}}}{G_{\text{max}}} \). The experimental \( \nu \) data at pressures \( p = 100 \) and 400 kPa is presented in Figure 13. It confirms the slight decrease of \( \nu \) with increasing confining pressure reported in [10]. Furthermore, for most tested materials a moderate increase of Poisson’s ratio with increasing void ratio was measured. In Figure 13, the prediction of Eq. (16) with \( G_{\text{max}} \) from Eqs. (9) to (12) and \( M_{\text{max}} \) from Eqs. (1) to (4) has been added as solid or dashed curves, respectively. The solid curves in Figure 13 have been obtained with \( C_u \) as input for the correlations while the dashed curves have been generated using \( C_{u,A} \). The prediction for both, \( C_u \) and \( C_{u,A} \), reflects the measured increase of \( \nu \) with decreasing pressure and increasing void ratio. For most tested sands, the predicted Poisson’s ratios are close to the measured data if \( C_u \) is used in the correlations. However, for some of the sands, in particular PL7, GG6 and S3, the \( \nu \) prediction is significantly improved if the correlations are applied with the average inclination \( C_{u,A} \).

6 Conclusions

Approx. 120 resonant column (RC) tests with additional P-wave measurements have been performed on 19 clean quartz sands with piecewise linear, gap-graded or smoothly shaped grain size distribution curves. For each material the P-wave velocity was measured at different densities and pressures. Similar as in an earlier test series on linear grain size distribution curves [10], a significant decrease of the constrained elastic modulus \( M_{\text{max}} \) with increasing uniformity coefficient \( C_u \) of the grain size distribution curve was also observed for the more complex gradations tested in the present study.

The applicability of the empirical equations (1) to (4) for \( M_{\text{max}} \) was inspected for the more complex grain size distribution curves. These correlations proposed by the authors in [10] are based on data for linear grain size distribution curves and consider the decrease of \( M_{\text{max}} \) with \( C_u \) for constant values of pressure and void ratio. Based on the present test data it could be demonstrated that the new correlations work well also for more complex grain size distribution curves. The prediction can be somewhat improved if the new correlations are applied with an average inclination \( C_{u,A} \) of the grain size distribution curve (see the scheme in Figure 9) instead of \( C_u \), in particular in the case of materials that are primarily composed of two components with significantly different grain size. For most practical cases, however, it may suffice to use the conventional \( C_u \) as input for the correlations.

A reanalysis of the data of small-strain shear modulus...
Fig. 13: Poisson’s ratio \( \nu \) at pressures \( p = 100 \) and 400 kPa for stepwise-linear (first row), gap-graded (second row) and smoothly shaped (third row) grain size distribution curves. The experimental data is compared to the Poisson’s ratio predicted by Eq. (16) with \( G_{\text{max}} \) from Eqs. (9) to (12) and \( M_{\text{max}} \) from Eqs. (1) to (4). The correlations have been applied either with \( C_u \) (solid curves) or \( C_{u,A} \) (dashed curves).

\( G_{\text{max}} \) and modulus degradation ratio \( G(\gamma)/G_{\text{max}} \) for the piecewise linear, gap-graded and smoothly shaped grain size distribution curves provided in [12] revealed that the \( G_{\text{max}} \) and \( G(\gamma)/G_{\text{max}} \) predictions for some of these sands can also be improved if the respective correlations derived from tests on linear gradations [9, 11] are applied with an average inclination \( C_{u,A} \) instead of \( C_u \). The same applies to Poisson’s ratio \( \nu \) which has been analyzed based on the measured \( M_{\text{max}} \) and \( G_{\text{max}} \) data.

Finally, it should be stressed that the extended empirical equations for \( M_{\text{max}} \), \( G_{\text{max}} \) and \( G(\gamma)/G_{\text{max}} \) are confirmed for clean sands with \( C_u \)-values less than 16 only. Therefore, until additional experimental data are available, they should be only applied within this range.

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References


