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Stiffness and damping of clean quartz sand with various grain size distribution curves

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Abstract: Approx. 240 resonant column (RC) tests have been performed on 26 clean quartz sands with piecewise linear, gap-graded, S-shaped or other smoothly shaped grain size distribution curves. For each material, the small-strain shear modulus $G_{\max}$, the shear modulus degradation curves $G(\gamma)/G_{\max}$ and the damping ratio curves $D(\gamma)$ were measured at different pressures and densities. The applicability of the extended empirical equations proposed by the authors in two previous publications, considering the influence of the uniformity coefficient $C_u = d_{60}/d_{10}$ of the grain size distribution curve, has been inspected for the various tested materials. These equations were originally developed based on RC test data for linear gradations. It is demonstrated that the extended empirical equations work well also for most of the "more complicated" grain size distribution curves tested in the present study.

CE Database subject headings: Small strain shear modulus; Shear modulus degradation; Damping ratio; Threshold shear strain amplitudes; Quartz sand; Grain size distribution curve; Uniformity coefficient; Resonant column tests

Introduction

For constant values of void ratio and pressure, the small-strain shear modulus $G_{\max}$ and the modulus degradation curves $G(\gamma)/G_{\max}$ of clean quartz sand depend strongly on the uniformity coefficient $C_u = d_{60}/d_{10}$ of the grain size distribution curve (Wichtmann & Triantafyllidis [5,6]). Higher C_u-values mean a lower small-strain stiffness and a more pronounced modulus degradation. In contrast, $G_{\max}$ and the curves $G(\gamma)/G_{\max}$ are rather independent of mean grain size d_{50}. Damping ratio $D(\gamma)$ is not significantly affected by a variation of both, d_{50} and C_u. The threshold shear strain amplitude indicating the transition from the linear elastic to the nonlinear elastic behaviour, $\gamma_{tl} = \gamma(G/G_{\max} = 0.99)$, was found to decrease slightly with increasing values of d_{50} and C_u [6]. In contrast, the threshold shear strain amplitude at the onset of settlement, γ_{tv}, was found almost independent of the grain size distribution curve. A micromechanical explanation of these experimental observations is provided in [5,6].

The common empirical formulas for $G_{\max}$ and $G(\gamma)/G_{\max}$ were developed based on tests on rather uniform sands. Therefore, they may overestimate the small-strain shear modulus and underestimate the shear modulus degradation of well-graded granular materials. In order to consider the influence of the uniformity coefficient, several empirical formulas have been extended by Wichtmann & Triantafyllidis [5,6]. For that purpose, correlations of the parameters of these equations with C_u have been formulated.

The extended empirical equations proposed by Wichtmann & Triantafyllidis [6] have been derived from resonant column (RC) tests performed on 25 clean quartz sands with linear grain size distribution curves (in the semi-logarithmic scale). It could be already demonstrated [5,6] that the prediction by the extended empirical equations agrees well with experimental data collected for various sands from the literature. However, most of the sands tested in the literature have almost linear or S-shaped grain size distribution curves. The present paper inspects whether the new correlations can be also applied to "more complicated" grain size distribution curves. For that purpose experimental data collected for stepwise linear (see grain size distribution curves in Figure 1a), gap-graded (Figure 1b), S-shaped and other smoothly shaped grain size distribution curves (Figure 1c) are analyzed. The test device and the testing procedure of the present study were the same as described by Wichtmann & Triantafyllidis [5,6].

Test results

Small strain shear modulus

In Figure 2 the $G_{\max}$ data measured for various materials at pressures $p = 100$ and 400 kPa are given as a function of void ratio e. The three rows of diagrams show data for stepwise linear, gap-graded or smoothly shaped grain size distribution curves, respectively. For all tested materials the well-known increase of $G_{\max}$ with increasing pressure and with decreasing void ratio was observed.

In Figure 3a the small-strain shear modulus at a void ratio $e = 0.55$ is plotted versus the uniformity coefficient $C_u = d_{60}/d_{10}$. The data are given for mean pressures $p =$ 100 and 400 kPa. Only materials with $G_{\max}$ data near $e =$ 0.55 have been considered in Figure 3a. The $G_{\max}(e = 0.55)$ values have been interpolated or carefully extrapolated. The data from the current tests (filled circles in Figure 3a) are compared with results obtained by Wichtmann & Triantafyllidis [5] for linear grain size distribution curves (open symbols in Figure 3a). The significant decrease of the small-strain shear modulus with increasing C_u observed by Wichtmann & Triantafyllidis [5] is confirmed by the present test series.

The shear moduli $G_{\max}(e)$ for $p = 100$ and 400 kPa predicted by Hardin's equation [1,3]

$$G_{\max} = A \frac{(a-e)^2}{1+e} (p_{\text{atm}})^{1-n} p^n \qquad (1)$$

with the correlations (2) to (4) proposed by Wichtmann &

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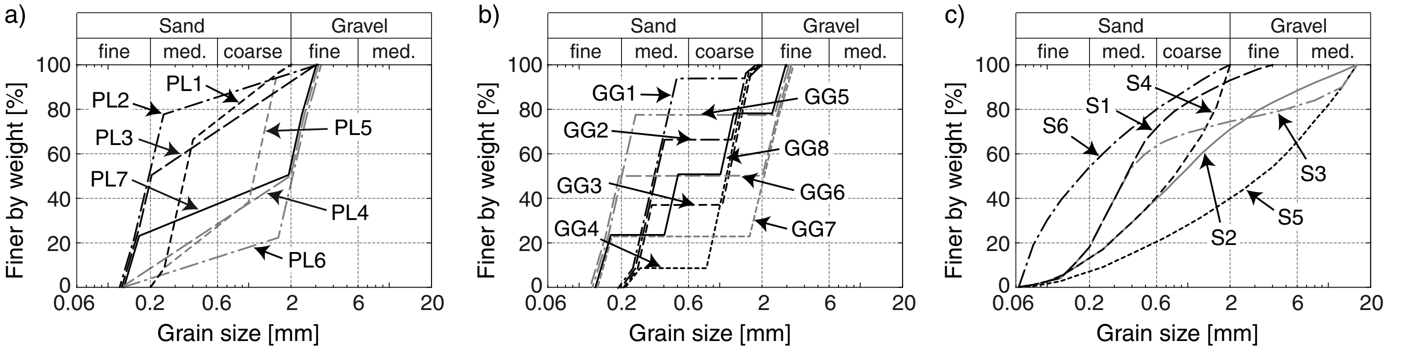

Fig. 1: *Tested grain size distribution curves: a) step-wise linear, b) gap-graded, c) smoothly shaped*

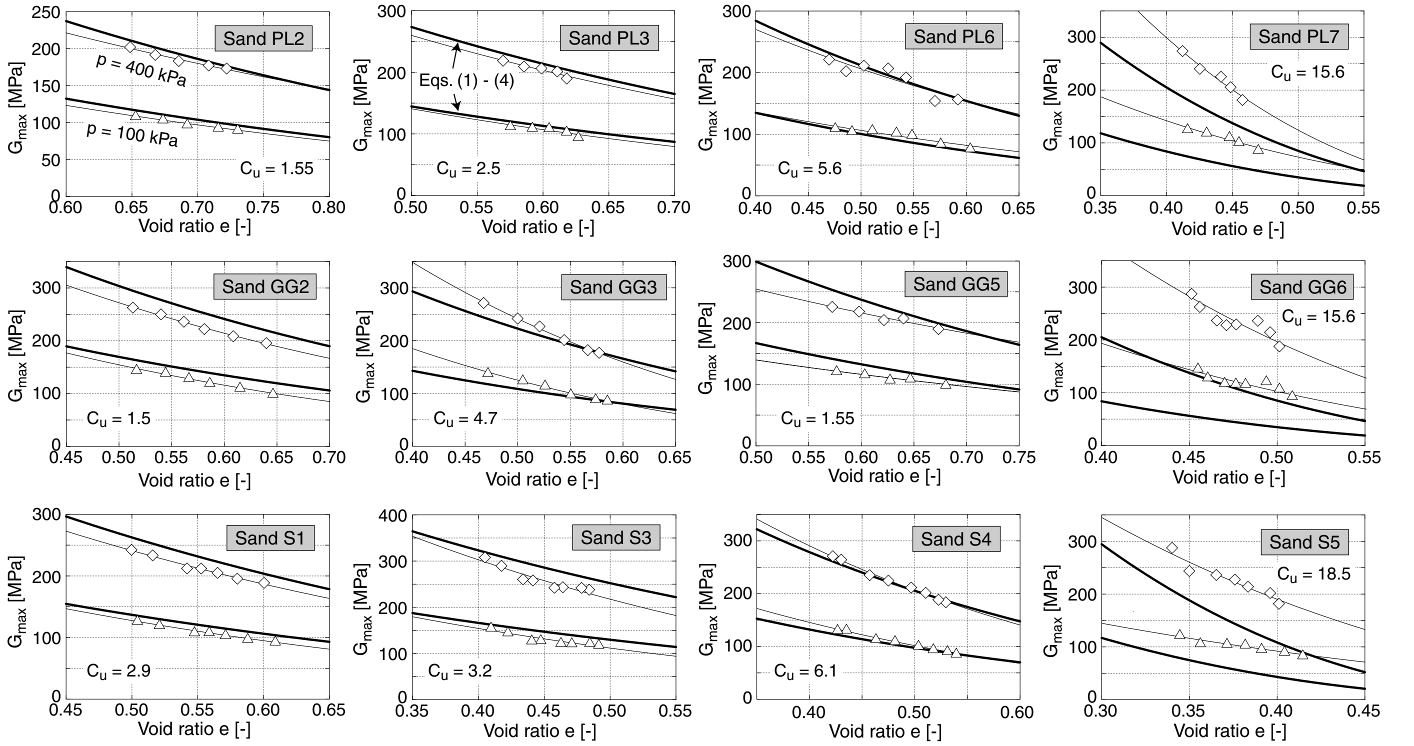

Fig. 2: *Small strain shear modulus* $G_{\max}(e)$ *for stepwise linear (first row), gap-graded (second row) and smoothly shaped (third row) grain size distribution curves. The experimental data is compared to* $G_{\max}$ *data predicted by Eqs. (1) to (4) (thick solid curves).*

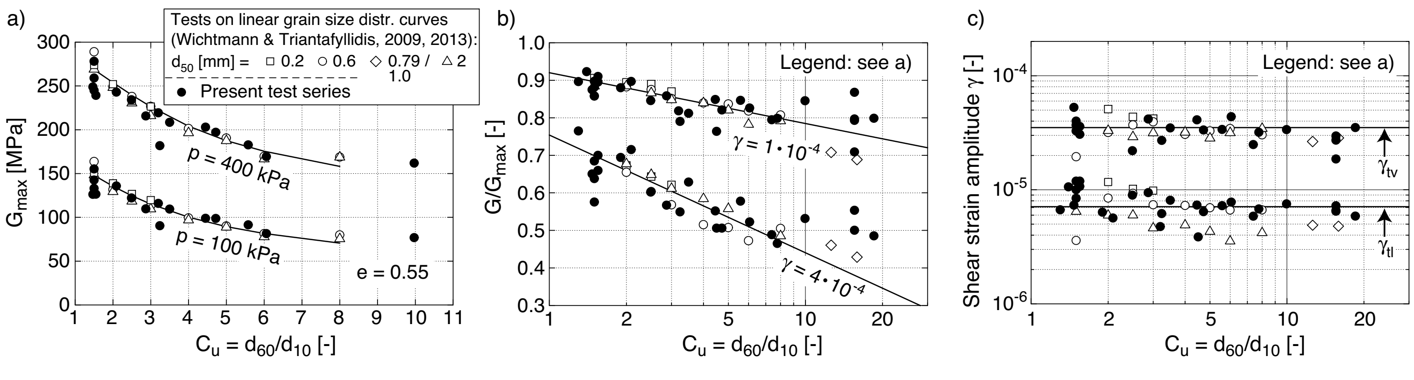

Fig. 3: *a) Small strain shear modulus* $G_{\max}(e = 0.55)$*, b) shear modulus degradation ratio* $G/G_{\max}$ *and c) threshold shear strain amplitudes* γ_{tl} *and* γ_{tv} *as a function of uniformity coefficient* $C_u = d_{60}/d_{10}$*. The data from the present test series are compared to data measured for linear grain size distribution curves (Wichtmann & Triantafyllidis [5, 6]).*

Triantafyllidis [5]

$$a = 1.94\ \exp(-0.066\ C_u) \quad (2)$$
$$n = 0.40\ {C_u}^{0.18} \quad (3)$$
$$A = 1563 + 3.13\ {C_u}^{2.98} \quad (4)$$

have been added as thick solid curves in Figure 2. For most of the "more complicated" grain size distribution curves, the experimental data are well approximated by Eqs. (1) to (4). The shear moduli of some materials (e.g. PL1, PL2, PL3, GG2, GG4, GG5, S1, S2, S3) are slightly overestimated while the shear moduli of some other materials (e.g. GG3, GG7) are slightly underestimated. However, for three tested materials (PL7, GG6 and S5) the experimental $G_{\max}$ data are significantly underestimated by the extended empirical equations (up to factor 2). These three materials either contain a large amount of gravel (about 60 % in the case of S5) or are primarily composed of two components with significantly different grain size (fine sand and fine gravel in case of PL7 and GG6). The unsatisfying prediction of Eqs. (1) to (4) for PL7 and GG6 may be tolerable since these grain size distribution curves are of limited practical relevance. The deviations between predicted and measured $G_{\max}$ values for S5 may be partly due to the high uniformity coefficient $C_u = 18.5$ which is beyond the range $1.5 \leq C_u \leq 16$ tested for the linear grain size distribution curves. The application of Eqs. (1) to (4) for arbitrary gradations should be thus restricted to the range $1.5 \leq C_u \leq 16$. The $G_{\max}$ values of well-graded granular materials with high gravel content (as in case of S5) need further experimental studies in future.

Letting away PL7, GG6 and S5, a quantitative analysis revealed that 67 or 93 %, respectively, of predicted $G_{\max}$ data differed either $\leq 10\%$ or $\leq 20\%$ from the measured data. For comparison, these values are 88 and 99 % for the linear grain size distribution curves tested by Wichtmann & Triantafyllidis [5]. In agreement with the results for linear gradations, the $G_{\max}$ data for the "more complicated" grain size distribution curves are predicted less accurate (deviation 55 or 82 %, respectively) if the following correlation

$$G_{\max} = 74000\ \frac{100 + D_r[\%]}{(1160 - D_r[\%])^2}\ {p_{\text{atm}}}^{1-0.48}\ p^{0.48} \quad (5)$$

with relative density $D_r = (e_{\max} - e)/(e_{\max} - e_{\min})$ proposed by Wichtmann & Triantafyllidis [5] is applied.

Modulus degradation curves

The shear modulus degradation curves $G(\gamma)/G_{\max}$ measured for various tested materials at pressures $p = 50$ and 400 kPa are given in Figure 4. In good agreement with the literature, for all tested materials the modulus degradation was found density-independent and stronger for lower pressures.

Figure 3b shows the shear modulus degradation ratio $G/G_{\max}$ as a function of C_u. The data are provided for a pressure $p = 400$ kPa and two different shear strain amplitudes $\gamma = 1 \times 10^{-4}$ and $\gamma = 4 \times 10^{-4}$. In accordance with Wichtmann & Triantafyllidis [6], independently of the shear strain amplitude, $G/G_{\max}$ decreases with increasing uniformity coefficient.

The modulus degradation curves predicted by the empirical formula proposed by Hardin & Drnevich [2]

$$\frac{G}{G_{\max}} = \frac{1}{1 + \frac{\gamma}{\gamma_r}\left[1 + a\ \exp\left(-\frac{\gamma}{\gamma_r}\right)\right]} \quad (6)$$

with the correlation developed by Wichtmann & Triantafyllidis [6]

$$a = 1.070\ \ln(C_u) \quad (7)$$

have been added as thick solid curves in Figure 4. The reference shear strain $\gamma_r = \tau_{\max}/G_{\max}$ has been evaluated using $\tau_{\max} = p\ \sin\varphi_P$ for isotropic stress conditions, applying the following correlation for the peak friction angle φ_P proposed by Wichtmann & Triantafyllidis [6]:

$$\varphi_P = 34.0^\circ\ \exp(0.27\ {D_r}^{1.8}) \quad (8)$$

For most of the tested grain size distribution curves, the experimental data are well approximated by Eqs. (6) to (8). However, for the materials PL7, GG6 and S5 the modulus degradation is underestimated.

A quantitative analysis of the data (without PL7, GG6 and S5) revealed that 83 or 99 %, respectively, of predicted $G(\gamma)/G_{\max}$ data differ either ≤ 0.05 or ≤ 0.1 from the measured data. For comparison, these values are 86 and 99 % for the linear grain size distribution curves tested by Wichtmann & Triantafyllidis [6]. If Eq. (6) is applied with a reference quantity $\sqrt{p/p_{\text{atm}}}$ instead of γ_r and with

$$a = 1093.7 + 1955.3\ \ln(C_u) \quad (9)$$

the percentage values are 65 and 92 % while 67 and 90 % are obtained for Stokoe's set of equations [4] in combination with the correlations proposed by Wichtmann & Triantafyllidis [6].

Damping ratio

The damping ratio data of the present test series is in good agreement with that obtained for linear grain size distribution curves (Wichtmann & Triantafyllidis [6]). No clear dependence of $D(\gamma)$ on d_{50} and C_u could be found. The correlations proposed by Wichtmann & Triantafyllidis [6] were found applicable also for the damping ratio data of the present study.

Threshold shear strain amplitudes

In Figure 3c mean values of the threshold shear strain amplitudes γ_{tl} and γ_{tv} measured in four tests on medium dense samples with pressures $p = 50$, 100, 200 and 400 kPa are plotted versus C_u. The γ_{tl} and γ_{tv} data collected for the "more complicated" grain size distribution curves agree well with the threshold amplitudes measured for the linear grain size distribution curves (Wichtmann & Triantafyllidis [6]). The threshold amplitudes are almost independent of C_u.

Summary and conclusions

Approx. 240 resonant column (RC) tests on 26 clean quartz sands with piecewise linear, gap-graded, S-shaped and other smoothly shaped grain size distribution curves have been performed. For each material the small-strain shear modulus $G_{\max}$, the modulus degradation curves $G(\gamma)/G_{\max}$ and the damping ratio curves $D(\gamma)$ were measured at different densities and pressures. In accordance

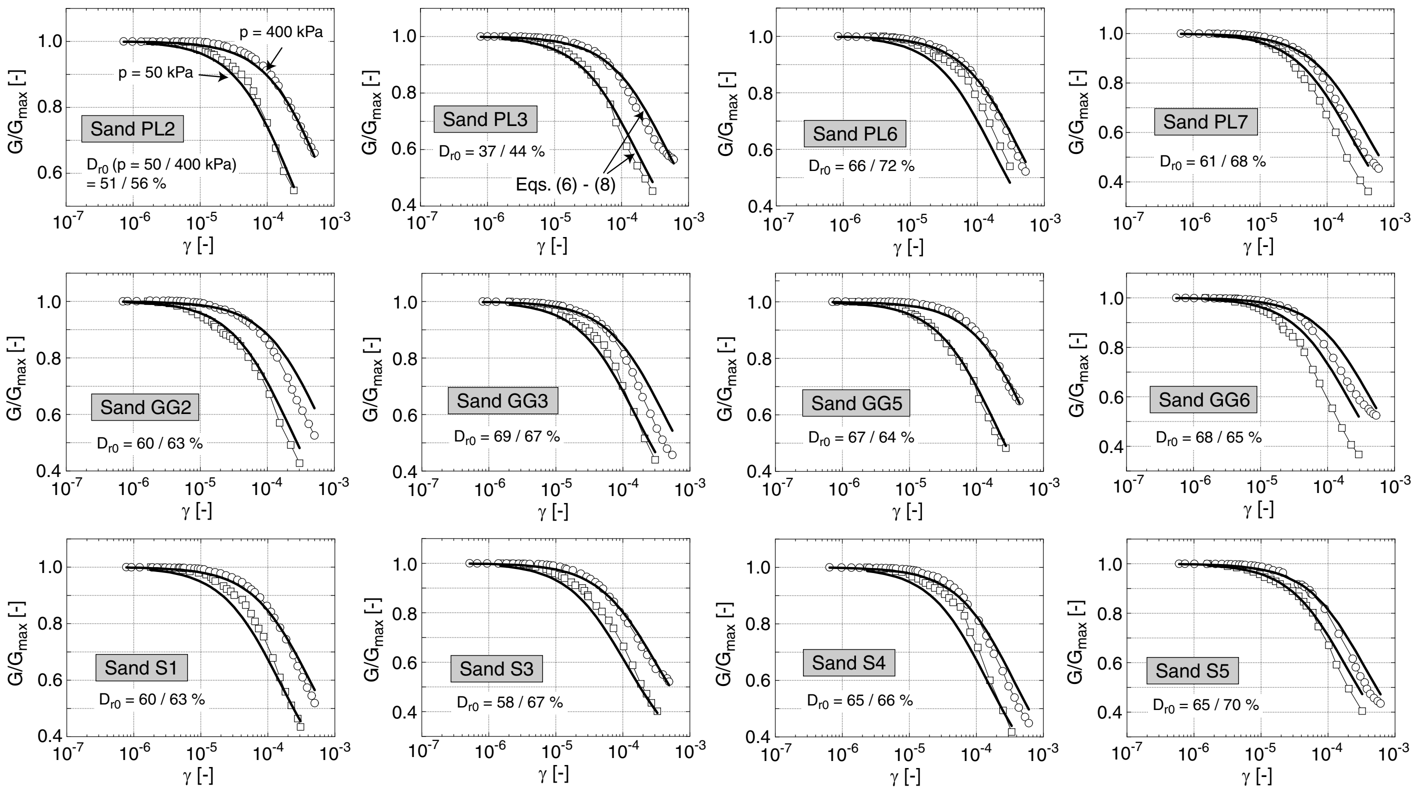

Fig. 4: *Shear modulus degradation curves $G(\gamma)/G_{\max}$ for stepwise-linear (first row), gap-graded (second row) and smoothly shaped (third row) grain size distribution curves. The experimental data is compared to $G(\gamma)/G_{\max}$ data predicted by Eqs. (6) to (8) (thick solid curves).*

with an earlier test series on linear grain size distribution curves (Wichtmann & Triantafyllidis [5, 6]), for constant values of void ratio and pressure a decrease of $G_{\max}$ with increasing uniformity coefficient $C_u = d_{60}/d_{10}$ of the tested grain size distribution curve was observed. Also the decrease of the shear modulus degradation ratio $G/G_{\max}$ with increasing C_u observed for linear grain size distribution curves is confirmed by the experimental data of the present study. In contrast, the damping ratio D and the threshold shear strain amplitudes γ_{tl} and γ_{tv} were found rather independent of C_u.

The applicability of several extended empirical equations proposed by Wichtmann & Triantafyllidis [5, 6] has been inspected based on the data of the present test series. In these empirical equations, the influence of the uniformity coefficient on $G_{\max}$ and $G(\gamma)/G_{\max}$ has been considered by using C_u-dependent parameters. The correlations between the parameters and C_u were originally developed based on RC tests performed on linear grain size distribution curves. Based on the present test data it could be demonstrated that the new correlations work well also for most of the "more complicated" grain size distribution curves.

Finally, it should be stressed that the extended empirical equations are confirmed for clean sands with C_u-values less than 16 only. Therefore, until additional experimental data are available, they should be only applied within this range.

Acknowledgements

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